Lecture Notes

on

Design of Machine elements

(Th.2)

5^{th} Semester, Mechanical Engg.

Prepared by

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TH.2 DESIGN OF MACHINE ELEMENTS

Name of the Course: Diploma in MECHANICAL ENGINEERING			
Course code:		**Semester**	**5th**
Total Period:	**60**	**Examination**	**3 hrs.**
Theory periods:	**4 P/W**	**I.A:**	**20**
Maximum marks:	**100**	**End Semester Examination:**	**80**

A. RATIONALE:

Machine design is the art of planning or devising new or improved machines to accomplish specific purposes. Idea of design is helpful in visualizing, specifying and selection of parts and components which constitute a machine. Hence all mechanical engineers should be conversant with the subject.

B. COURSE OBJECTIVES

At the end of the course the students will be able to

1. Understanding the behaviours of material and their uses.
2. Understanding the design of various fastening elements and their industrial uses.
3. Understanding the different failures of design elements.
4. Understanding the change of design to accomplish the different field of applications.
5. Design shafts, keys, couplings required for power transmission.
6. Design closed coil helical spring

C. CHAPTER WISE DISTRIBUTION OF PERIORDS

Sl.No.	Topic	Periods
01	INTRODUCTION	12
02	DESIGN OF FASTENING ELEMENTS	12
03	DESIGN OF SHAFT AND KEYS	12
04	DESIGN OF COUPLING	12
05	DESIGN OF CLOSED COIL HELICAL SPRING	12
	TOTAL	60

D. COURSE CONTENTS

1.0 Introduction:

1.1 Introduction to Machine Design and Classify it.
1.2 Different mechanical engineering materials used in design with their uses and their mechanical and physical properties.
1.3 Define working stress, yield stress, ultimate stress & factor of safety and stress –strain curve for M.S & C.I.
1.4 Modes of Failure (By elastic deflection, general yielding & fracture)
1.5 State the factors governing the design of machine elements.
1.6 Describe design procedure.

2.0 Design of fastening elements:

2.1 Joints and their classification.
2.2 State types of welded joints .
2.3 State advantages of welded joints over other joints.
2.4 Design of welded joints for eccentric loads.
2.5 State types of riveted joints and types of rivets.
2.6 Describe failure of riveted joints.
2.7 Determine strength & efficiency of riveted joints.
2.8 Design riveted joints for pressure vessel.
2.9 Solve numerical on Welded Joint and Riveted Joints.

3.0 Design of shafts and Keys:

3.1 State function of shafts.
3.2 State materials for shafts.
3.3 Design solid & hollow shafts to transmit a given power at given rpm based on
  a) Strength: (i) Shear stress, (ii) Combined bending tension;
  b) Rigidity: (i) Angle of twist, (ii) Deflection, (iii) Modulus of rigidity
3.4 State standard size of shaft as per I.S.
3.5 State function of keys, types of keys & material of keys.
3.6 Describe failure of key, effect of key way.
3.7 Design rectangular sunk key considering its failure against shear & crushing.
3.8 Design rectangular sunk key by using empirical relation for given diameter of shaft.
3.9 State specification of parallel key, gib-head key, taper key as per I.S.
3.10 Solve numerical on Design of Shaft and keys.

4.0 Design of Coupling:

4.1 Design of Shaft Coupling
4.2 Requirements of a good shaft coupling
4.3 Types of Coupling.
4.4 Design of Sleeve or Muff-Coupling.
4.5 Design of Clamp or Compression Coupling.
4.6 Solve simple numerical on above.

5.0 Design a closed coil helical spring:

5.1 Materials used for helical spring.
5.2 Standard size spring wire. (SWG).
5.3 Terms used in compression spring.
5.4 Stress in helical spring of a circular wire.
5.5 Deflection of helical spring of circular wire.
5.6 Surge in spring.
5.7 Solve numerical on design of closed coil helical compression spring.

Syllabus covered up to I.A-Chapters 1,2 &3

LEARNING RESOURCES

SL.NO	AUTHOR	TITLE OF THE BOOK	PUBLISHER
01	PANDYA AND SHAH	MACHINE DESIGN	CHAROTAR PP
02	R.S.KHURMI &J.K.GOPTA	A TEXT BOOK OF MACHINE DESIGN	S.CHAND
03	P.C.SHARMA &D.K AGRAWAL	A TEXT BOOK OF MACHINE DESIGN	S.K.KATARIYA
04	V.B.BHANDARI	DESIGNOF MACHINE ELEMENTS	TMH
05	S.MD.JALAUDEEN	DESIGN DATA BOOK	ANURADHA PUBLICATION

MACHINE DESIGN

Factors Governing the Design of Machine Elements:

a) Type of load and stresses caused by the load:

The load, on a machine component, may act in several ways due to which the internal stresses are set up.

b) Motion of the parts or kinematics of the machine:

The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required (Rectilinear motion, Curvilinear motion, Constant velocity, Constant or variable acceleration).

c) Selection of materials:

A designer should have a thorough knowledge of the properties of the materials and their behavior under working conditions.

d) Form and size of the parts:

The form and size are needed to be checked that the stresses induced in the designed cross-section are reasonably safe.

e) Frictional resistance and lubrication:

There is always a loss of power due to frictional resistance and it is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

f) Convenient and economical features:

- The operating features of the machine should be carefully studied so that the starting, controlling and stopping levers should be located on the basis of convenient handling.
- The economical operation of a machine which is to be used for production or for the processing of material should be studied.

g) Use of standard parts:

The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins..

h) Safety of operation:

Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator.

i) Workshop facilities:

A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop.

j) Number of machines to be manufactured:

The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design.

k) Cost of construction:

The aim of design engineer under all conditions, should be to reduce the manufacturing cost to the minimum.

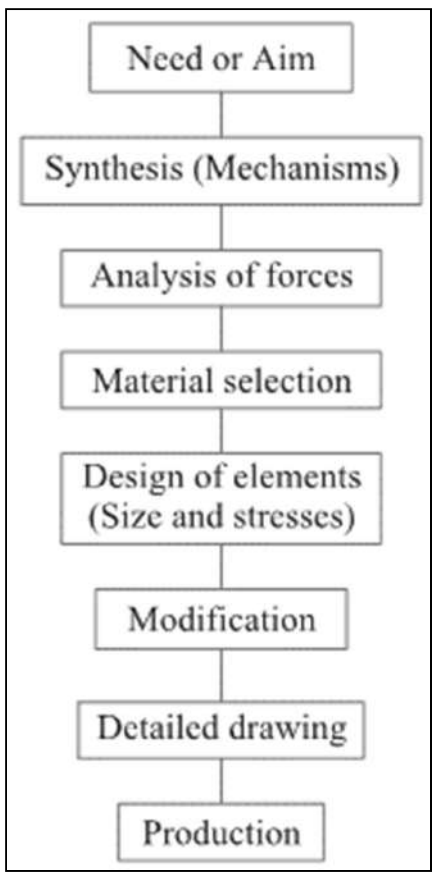
l) Assembling:

Every machine or structure must be assembled as a unit before it can function.

General design procedure:

i)Recognition of need:

Make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.



ii) Synthesis (Mechanisms)

Select the possible mechanism or group of mechanisms which will give the desired motion.

iii) Analysis of forces

Find the forces acting on each member of the machine and the energy transmitted by each member.

iv) Material selection

Select the material best suited for each member of the machine.

v) Design of elements (Size and Stresses)

Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used so that each member should not deflect or deform than the permissible limit.

vi) Modification

Modify the size of the member to agree with the past experience and judgment to facilitate manufacture and also by consideration of manufacturing to reduce overall cost.

vii) Detailed drawing

Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

viii) Production

The component, as per the drawing, is manufactured in the workshop.

Modes of Failure:

a) Failure by elastic deflection:

Two general types of excessive elastic deformation may occur

1. Excessive deflection under conditions of stable equilibrium, such as the deflection of a beam under gradually applied loads
2. Sudden deflection, or buckling, under conditions of unstable equilibrium.

Excessive elastic deformation of a machine part can mean failure of the machine just as much as if the part is completely fractured. For example, a shaft that is too flexible can cause rapid wear of the bearing, or the excessive deflection of closely mating parts can result in interference and damage to the parts.

The sudden buckling type of failure may occur in a slender column when the axial load exceeds the Euler critical load or when the external pressure acting against a thin-walled shell exceeds a critical value.

Failures due to excessive elastic deformation are controlled by the modulus of elasticity, not by the strength of the material.

b) Failure by general yielding:

Yielding, or excessive plastic deformation, occurs when the elastic limit of the metal has been exceeded.

Yielding produces a permanent change of shape, which may prevent the part from functioning properly any longer.

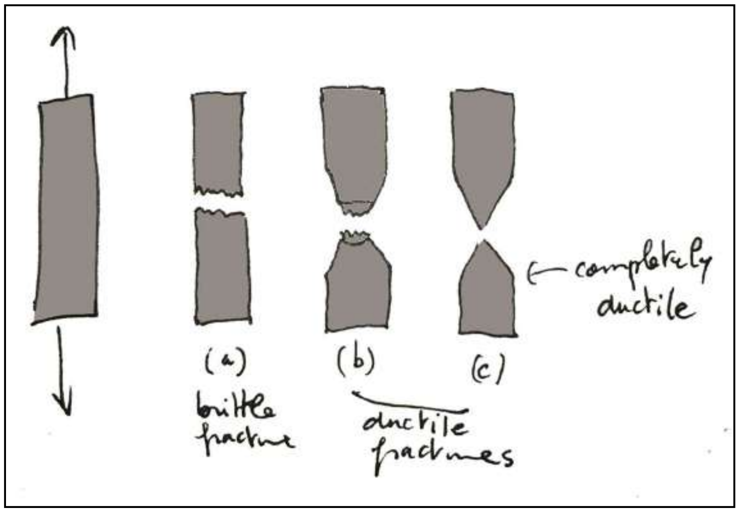
In a ductile metal under conditions of static loading at room temperature yielding rarely results in fracture, because the metal strain hardens as it deforms, and increased stress is required to produce further deformation.

Failure by excessive plastic deformation is controlled by the yield strength of the metal for a uniaxial condition of loading.

For more complex loading conditions the yield strength is still the significant parameter, but it must be used with a suitable failure criterion. At temperatures significantly greater than room temperature metals no longer exhibit strain hardening.

Instead, metals can continuously deform at constant stress in a time-dependent yielding known as creep.

The failure criterion under creep conditions is complicated by the fact that stress is not proportional to strain and the further fact that the mechanical properties of the material may change appreciably during service



c) Failure by fracture:

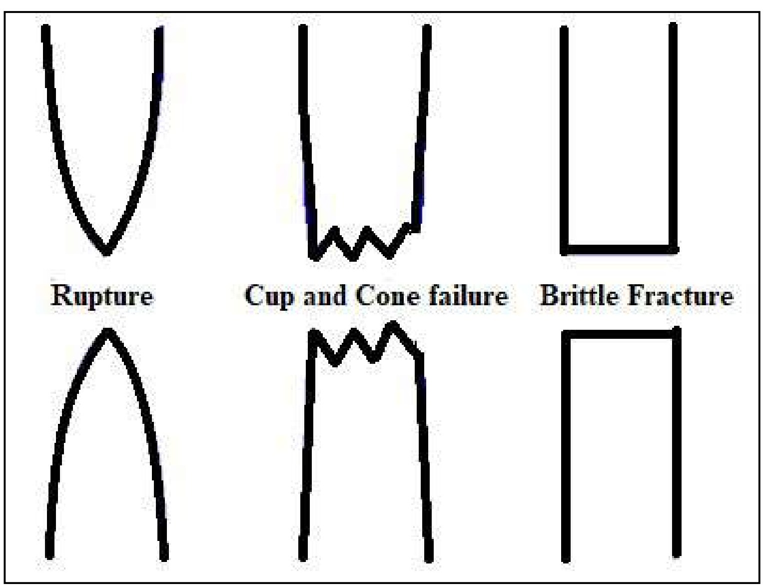
The formation of a crack that can result in complete disruption of the continuity of the member constitutes fracture.

A part made from a ductile metal that is loaded statically rarely fractures like a tensile specimen, because it will first fail by excessive plastic deformation.

However, metals fail by a fracture in three general ways:

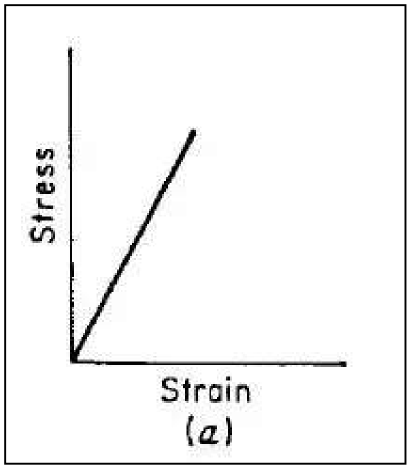
1. Sudden brittle fracture
2. Fatigue, or progressive fracture
3. Delayed fracture

The brittle material fractures under static loads with little outward evidence of yielding.



Cast Iron won't show any yield point phenomenon because they are so brittle in nature and when you apply load more than its elastic limiting load cast iron will be broken into pieces without any significant deformation.

Stress Vs Strain Curve for a Cast Iron looks like this

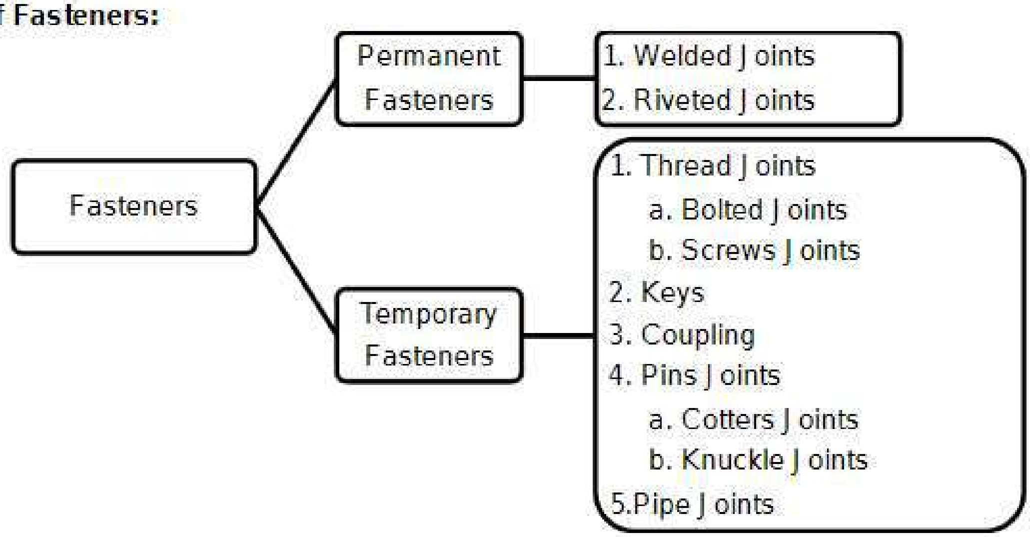


Chapter 02: Design of Fastening Elements

Joints:

- Joints are used to connect parts of a mechanism or machine.
- These mechanical joints can be temporary or permanent depending on whether the connection needs to be removed frequently or not removed at all.
- This determination is made by the designers and engineers of the machinery with the maintenance of the machinery taken into consideration.

Types of Fasteners:



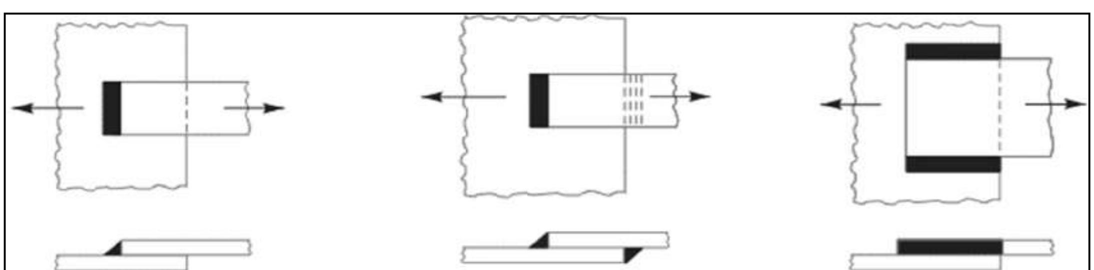
Welded Joint:

- A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material.
- The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding).

Types of Welded Joints:

a) Lap Joint:

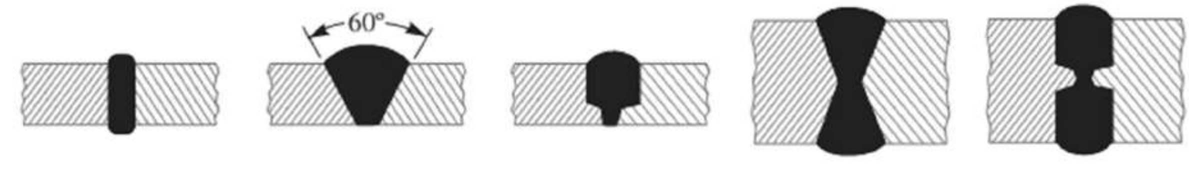
The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates and the cross-section of the fillet is approximately triangular.



(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

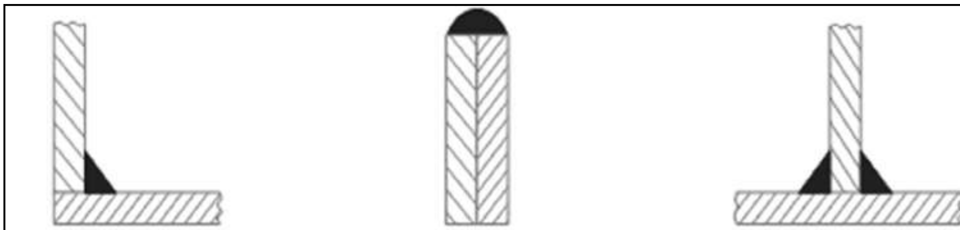
b) Butt Joint:

- The butt joint is obtained by placing the plates edge to edge as shown in Fig. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm.
- On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.



(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

The other type of welded joints are corner joint, edge joint and T-joint.



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

Advantages and Disadvantages of Welded Joints over Riveted Joints:

Advantages:

- The welded structures are usually lighter than riveted structures.
- provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
- Alterations and additions can be easily made in the existing structures.
- is smooth in appearance, therefore it looks pleasing.
- In welded connections, the tension members are not weakened as in the case of riveted joints.
- A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
- Sometimes, the members are of such a shape (i.e. circular steel pipes) that they afford difficulty for riveting, but they can be easily welded.
- The welding provides very rigid joints by providing rigid frames.
- It is possible to weld any part of a structure at any point, but riveting requires enough clearance.
- The process of welding takes less time than the riveting.

Disadvantages:

- Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
- It requires a highly skilled labor and supervision.
- Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
- The inspection of welding work is more difficult than riveting work.

Welded Joint with Eccentric Loads:

When the shear and bending stresses are simultaneously present in a joint, then maximum stresses are as follows:

Maximum normal stress,

$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

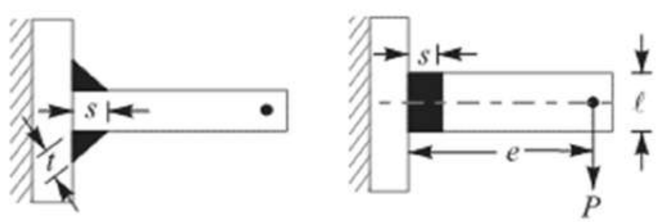
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

where σ_b = Bending stress, and

τ = Shear stress.

Case 1

Consider a T-joint fixed at one end and subjected to an eccentric load *P* at a distance *e*.



Let s = Size of weld, l = Length of weld, and t = Throat thickness.

The joint will be subjected to the following two types of stresses:

a) Direct shear stress due to the shear force P acting at the welds, and
b) Bending stress due to the bending moment P × e.

We know that area at the throat,

A = Throat thickness × Length of weld

= t × l × 2 = 2 t × l (For double fillet weld)

= 2 × 0.707 s × l = 1.414 s × l (so, t = s Cos 45° = 0.707 s)

∴ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\, s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \qquad \text{...(For both sides weld)}$$

$$= \frac{0.707\, s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

Bending moment, $M = P \times e$

∴ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\, P \times e}{s \times l^2}$

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

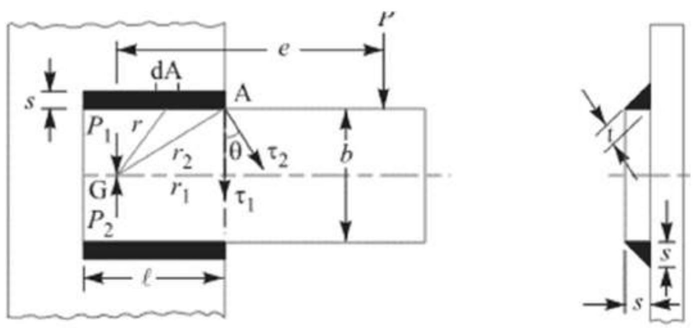
and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Case 2

When a welded joint is loaded eccentrically as shown in Fig., the following two types of the stresses are induced:

a) Direct or primary shear stress, and
b) Shear stress due to turning moment.



Let P = Eccentric load,

e = Eccentricity i.e. perpendicular distance between the line of action of load and centre of gravity (G) of the throat section or fillets,

l = Length of single weld,

s = Size or leg of weld, and

t = Throat thickness.

Let two loads P_1 and P_2 (each equal to P) are introduced at the centre of gravity 'G' of the weld system. The effect of load $P_1 = P$ is to produce direct shear stress which is assumed to be uniform over the entire weld length. The effect of load $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends of rotate the joint about the centre of gravity 'G' of the weld system. Due to the turning moment, secondary shear stress is induced

We know that the direct or primary shear stress,

$$\tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{2\,t \times l}$$

$$= \frac{P}{2 \times 0.707\, s \times l} = \frac{P}{1.414\, s \times l}$$

Since the shear stress produced due to the turning moment ($T = P \times e$) at any section is proportional to its radial distance from G, therefore stress due to $P \times e$ at the point A is proportional to AG (r_2) and is in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = \text{Constant}$$

or $$\tau = \frac{\tau_2}{r_2} \times r$$

where τ_2 is the shear stress at the maximum distance (r_2) and τ is the shear stress at any distance *r*.

Consider a small section of the weld having area *dA* at a distance *r* from G.

∴ Shear force on this small section = $\tau \times dA$

and turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2 \qquad \text{... [From equation } (i)\text{]}$$

∴ Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times dA \times r^2 = \frac{\tau_2}{r_2} \int dA \times r^2$$

$$= \frac{\tau_2}{r_2} \times J \qquad \left(\because J = \int dA \times r^2\right)$$

where J = Polar moment of inertia of the throat area about G.

∴ Shear stress due to the turning moment i.e. secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

∴ Resultant shear stress at A,

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

where θ = Angle between τ_1 and τ_2, and

$$\cos\theta = r_1 / r_2$$

Note: The polar moment of inertia of the throat area (A) about the centre of gravity (G) is obtained by the parallel axis theorem, i.e.

$$J = 2\,[I_{xx} + A \times x^2] \qquad \text{... (}\because\text{ of double fillet weld)}$$

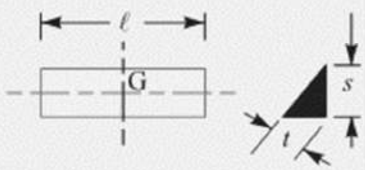
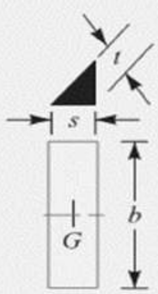
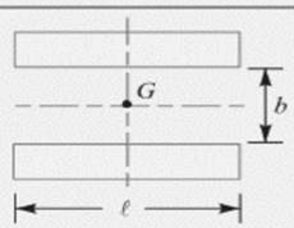
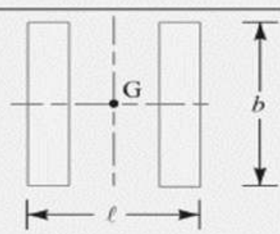
$$= 2\left[\frac{A \times l^2}{12} + A \times x^2\right] = 2A\left(\frac{l_2}{12} + x^2\right)$$

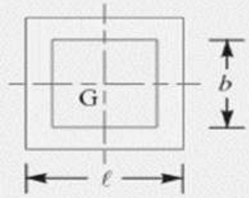
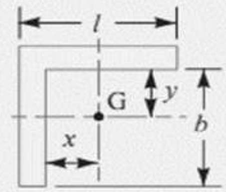
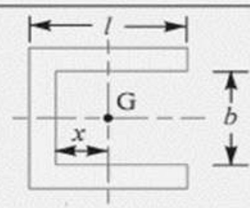
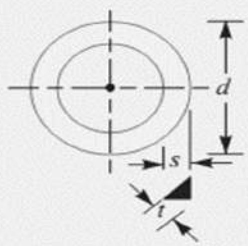
where A = Throat area = t × l = 0.707 s × l,

l = Length of weld, and

x = Perpendicular distance between the two parallel axes.

Table 10.7. Polar moment of inertia and section modulus of welds.

S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
1.		$\frac{t.l^3}{12}$	—
2.		$\frac{t.b^3}{12}$	$\frac{t.b^2}{6}$
3.		$\frac{t.l(3b^2 + l^2)}{6}$	$t.b.l$
4.		$\frac{t.b(b^2 + 3l^2)}{6}$	$\frac{t.b^2}{3}$

S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
5.		$\frac{t(b+l)^3}{6}$	$t\left(bl+\frac{b^2}{3}\right)$
6.	 $x=\frac{l^2}{2(l+b)},\ y=\frac{b^2}{2(l+b)}$	$t\left[\frac{(b+l)^4-6b^2l^2}{12(l+b)}\right]$	$t\left(\frac{4l.b+b^2}{6}\right)$ (Top) $t\left[\frac{b^2(4lb+b)}{6(2l+b)}\right]$ (Bottom)
7.	 $x=\frac{l^2}{2l+b}$	$t\left[\frac{(b+2l)^3}{12}-\frac{l^2(b+l)^2}{b+2l}\right]$	$t\left(l.b+\frac{b^2}{6}\right)$
8.		$\frac{\pi t d^3}{4}$	$\frac{\pi t d^2}{4}$

Note: In the above expressions, t is the throat thickness and s is the size of weld. It has already been discussed that $t = 0.707\ s$.

Example 10.9. *A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.*

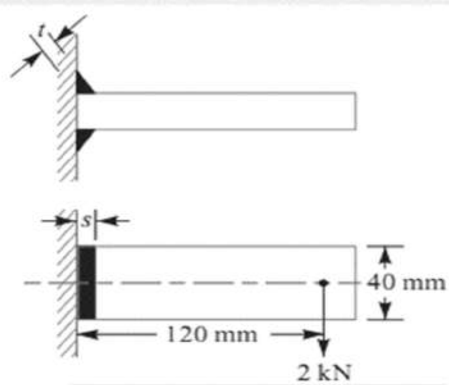


Fig. 10.24

Solution. Given: P = 2kN = 2000 N ; e = 120 mm ; l = 40 mm ; τ_{max} = 25 MPa = 25 N/mm^2

Let s = Size of weld in mm, and

t = Throat thickness.

The joint, as shown in Fig. 10.24, will be subjected to direct shear stress due to the shear force, P = 2000 N and bending stress due to the bending moment of $P \times e$.

We know that area at the throat,

$$A = 2t \times l = 2 \times 0.707\,s \times l$$
$$= 1.414\,s \times l$$
$$= 1.414\,s \times 40 = 56.56 \times s \text{ mm}^2$$

$\therefore$ Shear stress, $\tau = \dfrac{P}{A} = \dfrac{2000}{56.56 \times s} = \dfrac{35.4}{s}$ N/mm^2

Bending moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3$ N-mm

Section modulus of the weld through the throat,

$$Z = \frac{s \times l^2}{4.242} = \frac{s\,(40)^2}{4.242} = 377 \times s \text{ mm}^3$$

$$\therefore \text{ Bending stress, } \sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore$ $s = 320.3 / 25 = 12.8$ mm **Ans.**

Example 10.10. *A 50 mm diameter solid shaft is welded to a flat plate as shown in Fig. 10.25. If the size of the weld is 15 mm, find the maximum normal and shear stress in the weld.*

Solution. Given : D = 50 mm ; s = 15 mm ; P = 10 kN = 10 000 N ; e = 200 mm

Let t = Throat thickness.

The joint, as shown in Fig. 10.25, is subjected to direct shear stress and the bending stress. We know that the throat area for a circular fillet weld,

$$A = t \times \pi D = 0.707\, s \times \pi D = 0.707 \times 15 \times \pi \times 50 = 1666 \text{ mm}^2$$

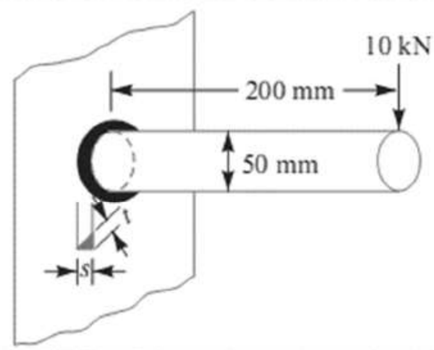


Fig. 10.25

$\therefore$ Direct shear stress,

$$\tau = \frac{P}{A} = \frac{10000}{1666} = 6 \text{ N/mm}^2 = 6 \text{ MPa}$$

We know that bending moment,

$$M = P \times e = 10\,000 \times 200 = 2 \times 10^6 \text{ N-mm}$$

From Table 10.7, we find that for a circular section, section modulus,

$$Z = \frac{\pi\, t\, D^2}{4} = \frac{\pi \times 0.707\, s \times D^2}{4} = \frac{\pi \times 0.707 \times 15\,(50)^2}{4} = 20\,825 \text{ mm}^3$$

$\therefore$ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{2 \times 10^6}{20\,825} = 96 \text{ N/mm}^2 = 96 \text{ MPa}$$

Maximum normal stress

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\,\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2} \times 96 + \frac{1}{2}\sqrt{(96)^2 + 4 \times 6^2}$$

$$= 48 + 48.4 = 96.4 \text{ MPa} \quad \textbf{Ans.}$$

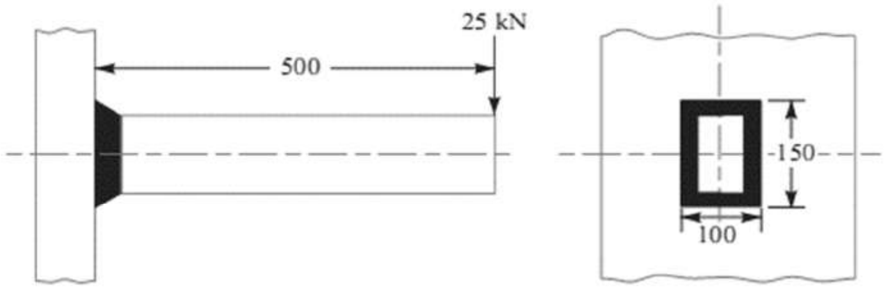
Maximum shear stress

We know that the maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(96)^2 + 4 \times 6^2} = 48.4 \text{ MPa} \quad \textbf{Ans.}$$

Example 10.11. *A rectangular cross-section bar is welded to a support by means of fillet welds as shown in Fig. 10.26.*

Determine the size of the welds, if the permissible shear stress in the weld is limited to 75 MPa.



All dimensions in mm

Fig. 10.26

Solution. Given : $P = 25$ kN $= 25 \times 10^3$N ; $\tau_{max} = 75$ MPa $= 75$N/mm^2 ; $l = 100$ mm ; $b = 150$ mm ; $e = 500$ mm

Let s = Size of the weld, and

t = Throat thickness.

The joint, as shown in Fig. 10.26, is subjected to direct shear stress and the bending stress. We know that the throat area for a rectangular fillet weld,

$$A = t\,(2b + 2l) = 0.707\,s\,(2b + 2l)$$
$$= 0.707s\,(2 \times 150 + 2 \times 100) = 353.5\,s \text{ mm}^2 \qquad \ldots (\because t = 0.707s)$$

$$\therefore \text{ Direct shear stress, } \tau = \frac{P}{A} = \frac{25 \times 10^3}{353.5\,s} = \frac{70.72}{s} \text{ N/mm}^2$$

We know that bending moment,

$$M = P \times e = 25 \times 10^3 \times 500 = 12.5 \times 10^6 \text{ N-mm}$$

From Table 10.7, we find that for a rectangular section, section modulus,

$$Z = t\left(b.l + \frac{b^2}{3}\right) = 0.707\,s\left[150 \times 100 + \frac{(150)^2}{3}\right] = 15\,907.5\,s \text{ mm}^3$$

$$\therefore \text{ Bending stress, } \sigma_b = \frac{M}{Z} = \frac{12.5 \times 10^6}{15\,907.5s} = \frac{785.8}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$75 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{785.8}{s}\right)^2 + 4\left(\frac{70.72}{s}\right)^2} = \frac{399.2}{s}$$

$$\therefore \quad s = 399.2 / 75 = 5.32 \text{ mm} \quad \textbf{Ans.}$$

Example 10.12. *An arm A is welded to a hollow shaft at section '1'. The hollow shaft is welded to a plate C at section '2'. The arrangement is shown in Fig. 10.27, along with dimensions. A force P = 15 kN acts at arm A perpendicular to the axis of the arm.*

Calculate the size of weld at section '1' and '2'. The permissible shear stress in the weld is 120 MPa.

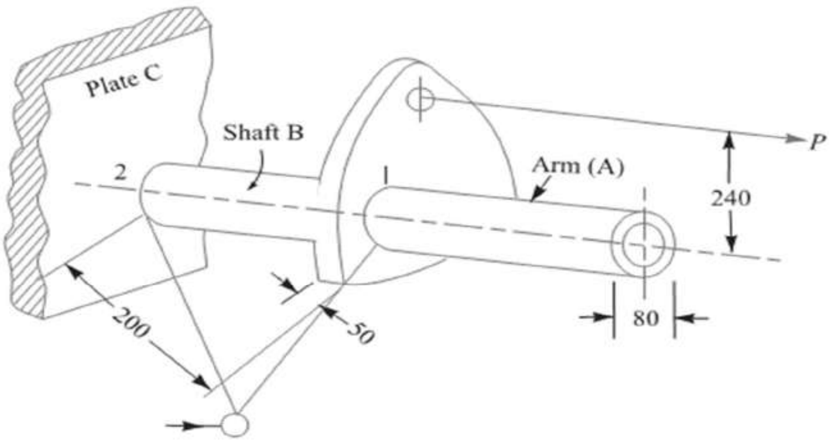


Fig. 10.27. All dimensions in mm.

Solution. Given : $P = 15 \text{ kN} = 15 \times 10^3 \text{ N}$; $\tau_{max} = 120 \text{ MPa} = 120 \text{ N/mm}^2$; $d = 80$ mm

Let s = Size of the weld.

The welded joint, as shown in Fig. 10.27, is subjected to twisting moment or torque (T) as well as bending moment (M).

We know that the torque acting on the shaft,

$$T = 15 \times 10^3 \times 240 = 3600 \times 10^3 \text{ N-mm}$$

$\therefore$ Shear stress,

$$\tau = \frac{2.83\, T}{\pi s d^2} = \frac{2.83 \times 3600 \times 10^3}{\pi \times s\ (80)^2} = \frac{506.6}{s} \text{ N/mm}^2$$

Bending moment,

$$M = 15 \times 10^3 \left(200 - \frac{50}{2}\right) = 2625 \times 10^3 \text{ N-mm}$$

$\therefore$ Bending stress,

$$\sigma_b = \frac{5.66\, M}{\pi s d^2} = \frac{5.66 \times 26.25 \times 10^3}{\pi s (80)^2} = \frac{738.8}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$120 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{738.8}{s}\right)^2 + 4\left(\frac{506.6}{s}\right)^2} = \frac{627}{s}$$

$\therefore$ $s = 627/120 = 5.2$ mm **Ans.**

Example 10.13. *A bracket carrying a load of 15 kN is to be welded as shown in Fig. 10.28. Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.*

Solution. Given : $P = 15 \text{ kN} = 15 \times 10^3 \text{ N}$; $\tau = 80 \text{ MPa} = 80 \text{ N/mm}^2$; $b = 80$ mm ; $l = 50$ mm; $e = 125$ mm

Let s = Size of weld in mm, and

t = Throat thickness.

We know that the throat area,

$$A = 2 \times t \times l = 2 \times 0.707\, s \times l$$
$$= 1.414\, s \times l = 1.414 \times s \times 50 = 70.7\, s \text{ mm}^2$$

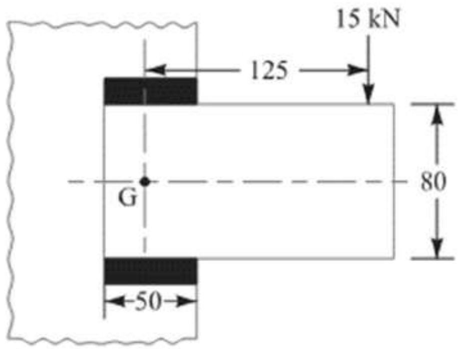
$\therefore$ Direct or primary shear stress,

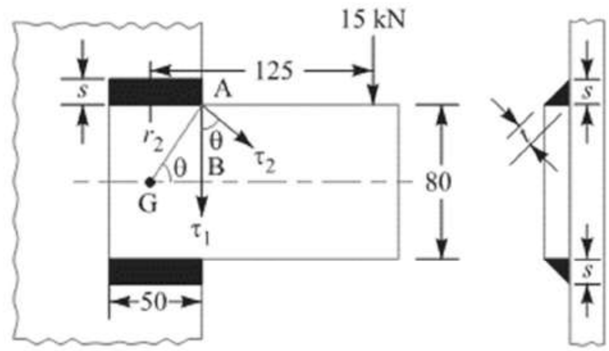
$$\tau_1 = \frac{P}{A} = \frac{15 \times 10^3}{70.7\, s} = \frac{212}{s} \text{ N/mm}^2$$

From Table 10.7, we find that for such a section, the polar moment of inertia of the throat area of the weld about G is

$$J = \frac{t.l\,(3b^2 + l^2)}{6} = \frac{0.707\ s \times 50\,[3\,(80)^2 + (50)^2]}{6}\ \text{mm}^4$$

$$= 127\,850\ s\ \text{mm}^4 \qquad \ldots (\because\ t = 0.707\ s)$$





All dimensions in mm.

Fig. 10.28 **Fig. 10.29**

From Fig. 10.29, we find that $AB = 40$ mm and $BG = r_1 = 25$ mm.

$\therefore$ Maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(40)^2 + (25)^2} = 47\ \text{mm}$$

Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{15 \times 10^3 \times 125 \times 47}{127\,850\ s} = \frac{689.3}{s}\ \text{N/mm}^2$$

and $$\cos\theta = \frac{r_1}{r_2} = \frac{25}{47} = 0.532$$

We know that resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2\ \cos\theta}$$

$$80 = \sqrt{\left(\frac{212}{s}\right)^2 + \left(\frac{689.3}{s}\right)^2 + 2 \times \frac{212}{s} \times \frac{689.3}{s} \times 0.532} = \frac{822}{s}$$

$\therefore$ $s = 822 / 80 = 10.3$ mm **Ans.**

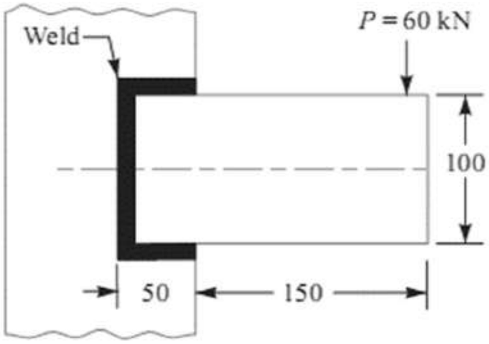
Example 10.14. *A rectangular steel plate is welded as a cantilever to a vertical column and supports a single concentrated load P, as shown in Fig. 10.30.*

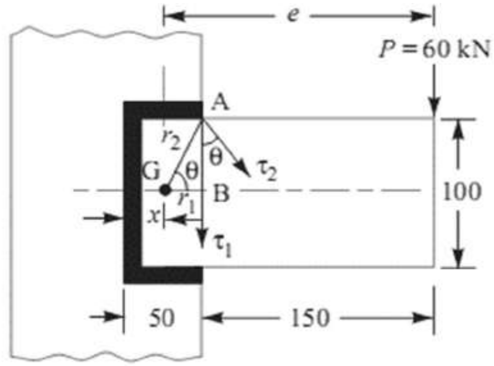
Determine the weld size if shear stress in the same is not to exceed 140 MPa.

Solution. Given : $P = 60$ kN $= 60 \times 10^3$ N ; $b = 100$ mm ; $l = 50$ mm ; $\tau = 140$ MPa $= 140$ N/mm^2

Let s = Weld size, and

t = Throat thickness.





All dimensions in mm.

Fig. 10.30 **Fig. 10.31**

First of all, let us find the centre of gravity (G) of the weld system, as shown in Fig. 10.31.

Let x be the distance of centre of gravity (G) from the left hand edge of the weld system. From Table 10.7, we find that for a section as shown in Fig. 10.31,

$$x = \frac{l^2}{2l + b} = \frac{(50)^2}{2 \times 50 + 100} = 12.5 \text{ mm}$$

and polar moment of inertia of the throat area of the weld system about G,

$$J = t\left[\frac{(b+2l)^3}{12} - \frac{l^2\,(b+l)^2}{b+2l}\right]$$

$$= 0.707\,s\left[\frac{(100 + 2\times 50)^3}{12} - \frac{(50)^2\,(100+50)^2}{100 + 2\times 50}\right] \quad \ldots (\because\ t = 0.707\ s)$$

$$= 0.707s\,[670 \times 10^3 - 281 \times 10^3] = 275 \times 10^3\ s \text{ mm}^4$$

Distance of load from the centre of gravity (G) *i.e.* eccentricity,

$$e = 150 + 50 - 12.5 = 187.5 \text{ mm}$$

$$r_1 = BG = 50 - x = 50 - 12.5 = 37.5 \text{ mm}$$

$$AB = 100 / 2 = 50 \text{ mm}$$

We know that maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(50)^2 + (37.5)^2} = 62.5 \text{ mm}$$

$$\therefore \qquad \cos\theta = \frac{r_1}{r_2} = \frac{37.5}{62.5} = 0.6$$

We know that throat area of the weld system,

$$A = 2 \times 0.707s \times l + 0.707s \times b = 0.707\ s\,(2l + b)$$

$$= 0.707s\,(2 \times 50 + 100) = 141.4\ s \text{ mm}^2$$

$\therefore$ Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{60 \times 10^3}{141.4s} = \frac{424}{s} \text{ N/mm}^2$$

and shear stress due to the turning moment or secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{60 \times 10^3 \times 187.5 \times 62.5}{275 \times 10^3\ s} = \frac{2557}{s} \text{ N/mm}^2$$

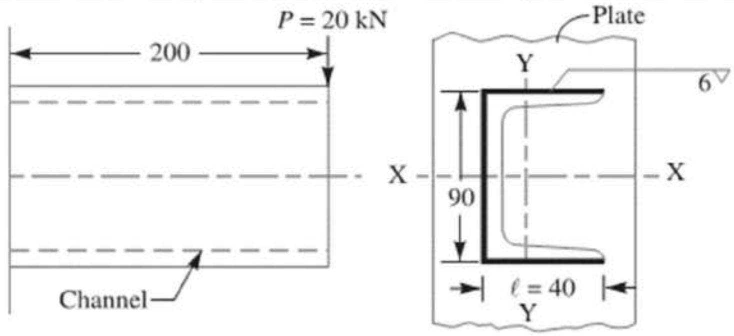
We know that the resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2 \times \cos\theta}$$

$$140 = \sqrt{\left(\frac{424}{s}\right)^2 + \left(\frac{2557}{s}\right)^2 + 2\times\frac{424}{s}\times\frac{2557}{s}\times 0.6} = \frac{2832}{s}$$

$\therefore$ $s = 2832 / 140 = 20.23$ mm **Ans.**

Example 10.15. *Find the maximum shear stress induced in the weld of 6 mm size when a channel, as shown in Fig. 10.32, is welded to a plate and loaded with 20 kN force at a distance of 200 mm.*



All dimensions in mm.

Fig. 10.32

Solution. Given : $s = 6$ mm ; $P = 20$ kN $= 20 \times 10^3$ N ; $l = 40$ mm ; $b = 90$ mm

Let t = Throat thickness.

First of all, let us find the centre of gravity (G) of the weld system as shown in Fig. 10.33. Let x be the distance of centre of gravity from the left hand edge of the weld system. From Table 10.7, we find that for a section as shown in Fig. 10.33,

$$x = \frac{l^2}{2l + b} = \frac{(40)^2}{2\times 40 + 90} = 9.4 \text{ mm}$$

and polar moment of inertia of the throat area of the weld system about G,

$$J = t\left[\frac{(b+2l)^3}{12} - \frac{l^2\,(b+l)^2}{b+2l}\right]$$

$$= 0.707\,s\left[\frac{(90+2\times 40)^3}{12} - \frac{(40)^2\,(90+40)^2}{90+2\times 40}\right] \quad \ldots (\because\ t = 0.707\,s)$$

$$= 0.707 \times 6\,[409.4 \times 10^3 - 159 \times 10^3] = 1062.2 \times 10^3 \text{ mm}^4$$

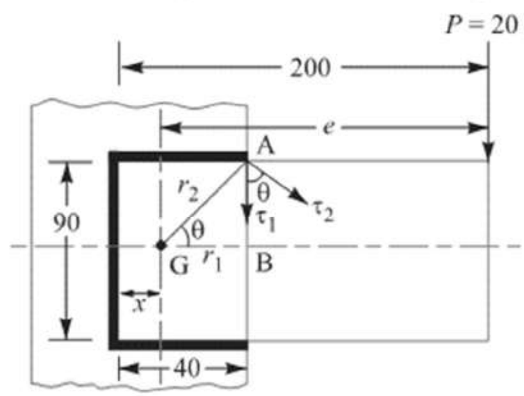


Fig. 10.33

Distance of load from the centre of gravity (G), *i.e.* eccentricity,

$$e = 200 - x = 200 - 9.4 = 190.6 \text{ mm}$$

$$r_1 = BG = 40 - x = 40 - 9.4 = 30.6 \text{ mm}$$

$$AB = 90/2 = 45 \text{ mm}$$

We know that maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(45)^2 + (30.6)^2} = 54.4 \text{ mm}$$

$$\therefore \quad \cos\theta = \frac{r_1}{r_2} = \frac{30.6}{54.4} = 0.5625$$

We know that throat area of the weld system,

$$A = 2 \times 0.707s \times l + 0.707s \times b = 0.707\, s\,(2l + b)$$

$$= 0.707 \times 6\,(2 \times 40 + 90) = 721.14 \text{ mm}^2$$

$\therefore$ Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{20 \times 10^3}{721.14} = 27.7 \text{ N/mm}^2$$

and shear stress due to the turning moment or secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{20 \times 10^3 \times 190.6 \times 54.4}{1062.2 \times 10^3} = 195.2 \text{ N/mm}^2$$

We know that resultant or maximum shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

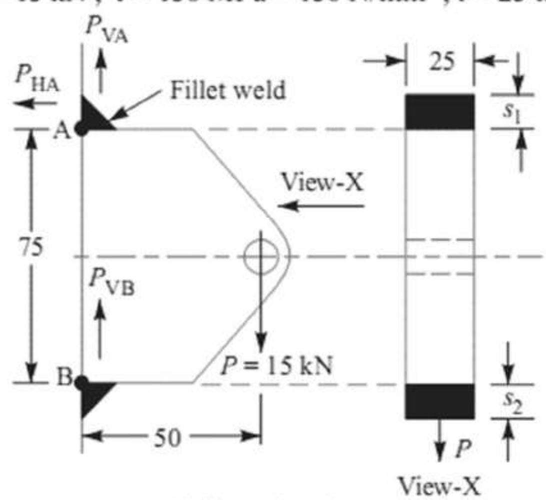
$$= \sqrt{(27.7)^2 + (195.2)^2 + 2 \times 27.7 \times 195.2 \times 0.5625}$$

$$= 212 \text{ N/mm}^2 = 212 \text{ MPa} \quad \textbf{Ans.}$$

Example 10.16. *The bracket, as shown in Fig. 10.34, is designed to carry a dead weight of $P = 15$ kN.*

What sizes of the fillet welds are required at the top and bottom of the bracket? Assume the forces act through the points A and B. The welds are produced by shielded arc welding process with a permissible strength of 150 MPa.

Solution. Given : $P = 15$ kN ; $\tau = 150$ MPa $= 150$ N/mm^2 ; $l = 25$ mm



All dimensions in mm.

Fig. 10.34

In the joint, as shown in Fig. 10.34, the weld at A is subjected to a vertical force P_{VA} and a horizontal force P_{HA}, whereas the weld at B is subjected only to a vertical force P_{VB}. We know that

$$P_{VA} + P_{VB} = P \quad \text{and} \quad P_{VA} = P_{VB}$$

∴ Vertical force at A and B,

$$P_{VA} = P_{VB} = P/2 = 15/2 = 7.5 \text{ kN} = 7500 \text{ N}$$

The horizontal force at A may be obtained by taking moments about point B.

$$\therefore \quad P_{HA} \times 75 = 15 \times 50 = 750$$

or

$$P_{HA} = 750/75 = 10 \text{ kN}$$

Size of the fillet weld at the top of the bracket

Let s_1 = Size of the fillet weld at the top of the bracket in mm.

We know that the resultant force at A,

$$P_A = \sqrt{(P_{VA})^2 + (P_{HA})^2} = \sqrt{(7.5)^2 + (10)^2} = 12.5 \text{ kN} = 12\,500 \text{ N} \quad ...(i)$$

We also know that the resultant force at A,

$$P_A = \text{Throat area} \times \text{Permissible stress}$$

$$= 0.707\, s_1 \times l \times \tau = 0.707\, s_1 \times 25 \times 150 = 2650\, s_1 \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$s_1 = 12\,500 / 2650 = 4.7 \text{ mm } \textbf{Ans.}$$

Size of fillet weld at the bottom of the bracket

Let s_2 = Size of the fillet weld at the bottom of the bracket.

The fillet weld at the bottom of the bracket is designed for the vertical force (P_{VB}) only. We know that

$$P_{VB} = 0.707\, s_2 \times l \times \tau$$

$$7500 = 0.707\, s_2 \times 25 \times 150 = 2650\, s_2$$

$$\therefore \quad s_2 = 7500 / 2650 = 2.83 \text{ mm } \textbf{Ans.}$$

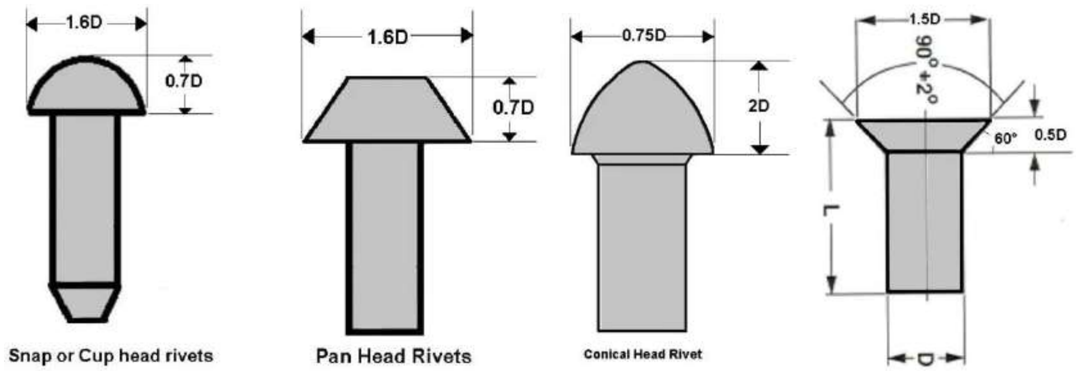
Types of Rivets

Following are the different types of Rivets:

1. Snap head or cup head rivets
2. Pan head rivets
3. Conical head rivets
4. Countersunk head rivets
5. Flathead rivets
6. Buffercated head rivet
7. Hollow head rivets.

1. Snap head or cup head rivets

Rivets with this kind of heads are used most of all. This type of rivets is shown in the figure. The head is of a semi-circle in shape. Its diameter is 1.6D and its height is 0.7D. The joints of this rivet are very strong. That is why it is widely used in bridges made of iron material.



2. Pan head rivets

In heavy engineering, pan head rivets are used. These have been shown in the figure. The height of its head is 0.7D and its diameter is 1.6D. The upper portion of the rivet head is flat and taper. Small diameter of the head is equal to the diameter of the rivet.

3. Conical Head Rivet

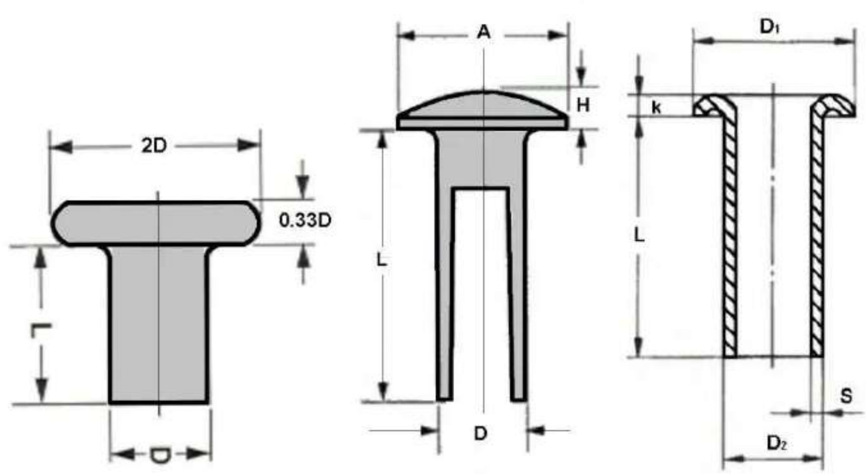
The conical head rivet is shown in the figure. The conical shape is given is used for light jobs. A conical shape is given to the head by a hammer. The diameter of its head is 0.75D and the height of its head is 2D.

4. Counter Sunk Head Rivet

At places where it is necessary to keep the surface plane even after fixing a rivet, this type of rivets is used. The diameter of its head is 1.5D, taper idle is 0.5D and taper angle is 60 . There are three kinds as shown in the figure.

5. Flat Head Rivet

For small and light jobs o sheet metal, flat head rivets are used. These are generally used in non-ferrous metals and thin sheets. Its head is flat. The diameter of its head is 2D and its height is 0.33D. This rivet is shown in the figure.



6. Buffercated Rivet

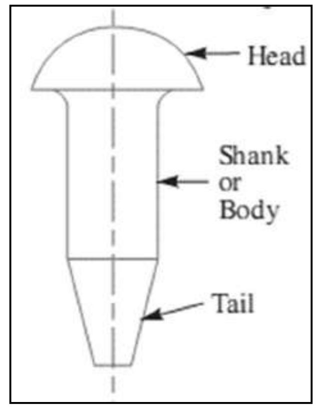
These types of rivets are different from other rivets. These are used for joining chains etc. in place of pins.

7. Hollow Rivet

Hollow rivets used where a part of the machine moves and it is also necessary to keep this part attached to the machine.

Riveted Joint:

- A rivet is a short cylindrical bar with a head integral to it.
- The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail.
- The riveted joints are widely used for joining light metals.



Types of Riveted Joints:

a) Lap Joint:

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

b) Butt Joint:

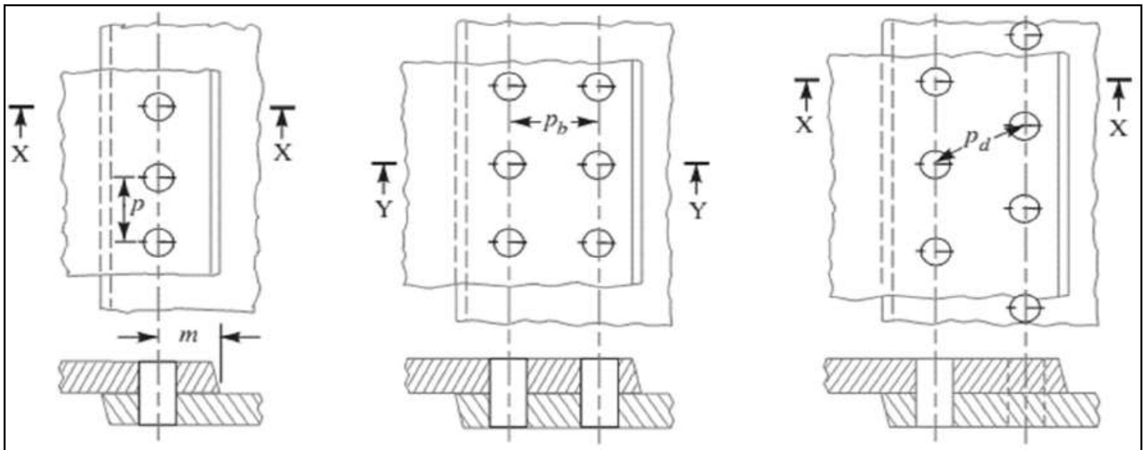
A butt joint is that in which the main plates are kept in alignment butting (i.e. touching) each other and a cover plate (i.e. strap) is placed either on one side or on both sides of the main plates.

Butt joints are of the following two types:

In a ***single strap butt joint,*** the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

In a ***double strap butt joint,*** the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

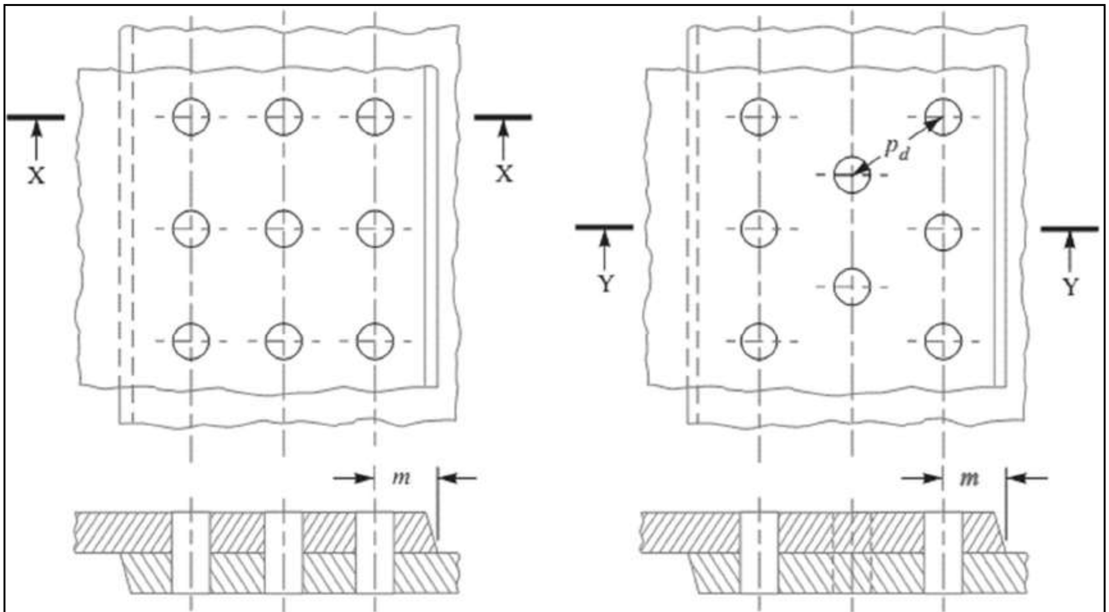
Similarly the joints may be ***triple riveted*** or ***quadruple riveted***.



(*a*) Single riveted lap joint. (*b*) Double riveted lap joint (Chain riveting). (*c*) Double riveted lap joint (Zig-zag riveting).

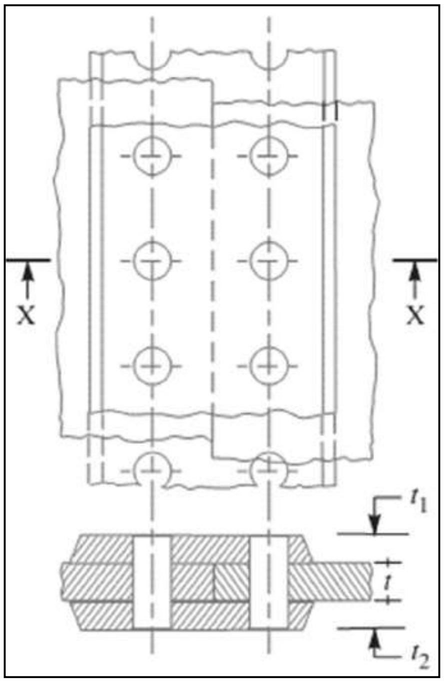
Single and double riveted lap joints.

When the rivets in the various rows are opposite to each other, then the joint is said to be ***chain riveted***. On the other hand, if the rivets in the adjacent rows are staggered in such a way that every rivet is in the middle of the two rivets of the opposite row, then the joint is said to be ***zig-zag riveted***.

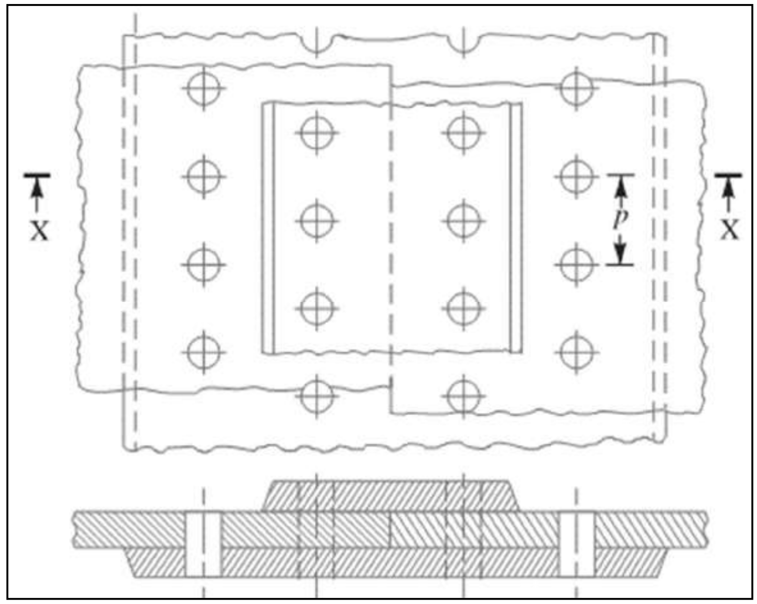


(*a*) Chain riveting. (*b*) Zig-zag riveting.

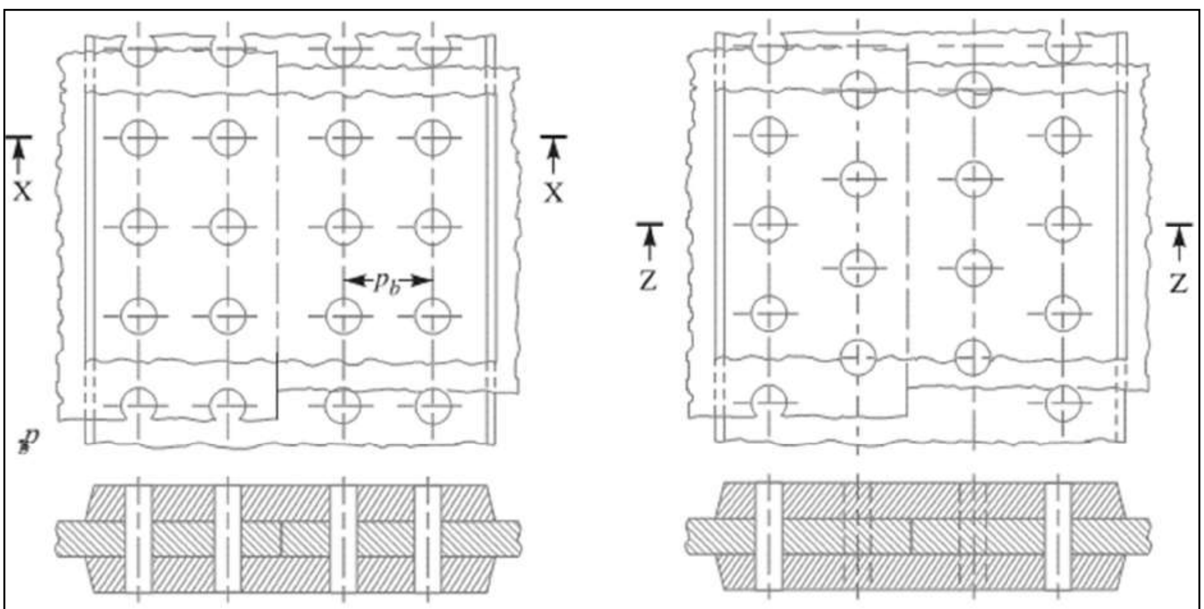
Triple riveted lap joint.



Single riveted double strap butt joint.



Double riveted double strap (unequal) butt joint with zig-zag riveting.



(*a*) Chain riveting. (*b*) Zig-zag riveting.

Double riveted double strap (equal) butt joints.

Important Terms:

a) **Pitch:** It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig. 9.6. It is usually denoted by *p*.

b) **Back pitch:** It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. 9.6. It is usually denoted by p_b.

c) **Diagonal pitch:** It is the distance between the centers of the rivets in adjacent rows of zig-zag riveted joint as shown in Fig. 9.6. It is usually denoted by p_d.

d) **Margin or marginal pitch:** It is the distance between the centre of rivet hole to the nearest edge of the plate as shown in Fig. 9.6. It is usually denoted by m.

Failures of Riveted Joints:

1) Tearing of the plate at an edge:

A joint may fail due to tearing of the plate at an edge which can be avoided by keeping the margin, m = 1.5d, where d is the diameter of the rivet hole.

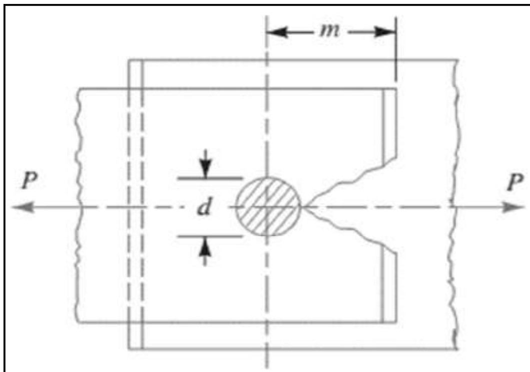


Fig. 9.13. Tearing of the plate at an edge.

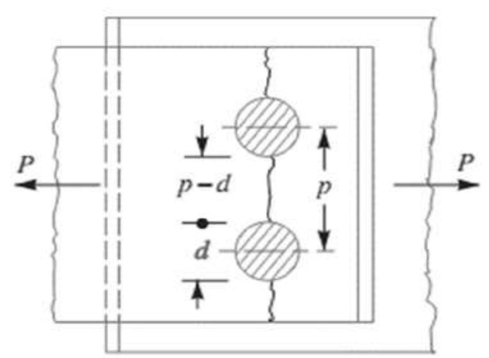


Fig. 9.14. Tearing of the plate across the rows of rivets.

2) Tearing of the plate across a row of rivets:

Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. 9.14. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate.

Let p = Pitch of the rivets, d = Diameter of the rivet hole, t = Thickness of the plate, and σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

$$A_t = (p - d)\, t$$

∴ Tearing resistance or pull required to tear off the plate per pitch length,

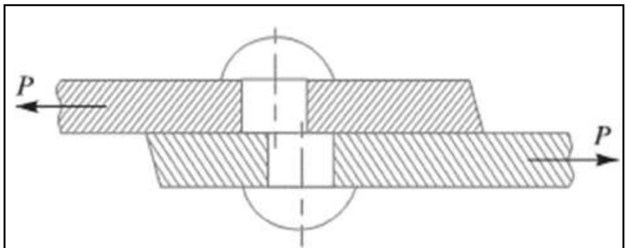
$$P_t = A_t.\, \sigma_t = (p - d)\, t\, .\sigma_t$$

When the tearing resistance (P_t) is greater than the applied load (P) per pitch length, then this type of failure will not occur.

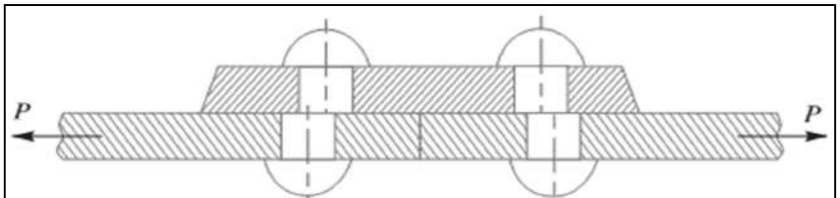
3) Shearing of the rivets:

The plates which are connected by the rivets exert tensile stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig. 9.15.

It may be noted that the rivets are in single shear in a lap joint and in a single cover butt joint, as shown in Fig. 9.15. But the rivets are in double shear in a double cover butt joint as shown in Fig. 9.16. The resistance offered by a rivet to be sheared off is known as shearing resistance or shearing strength or shearing value of the rivet.



(*a*) Shearing off a rivet in a lap joint.



(*b*) Shearing off a rivet in a single cover butt joint.

Fig. 9.15. Shearing of rivets.

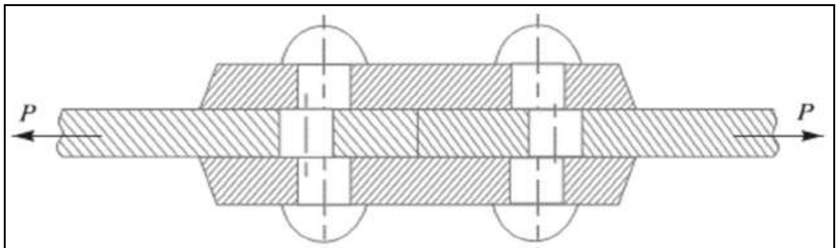


Fig. 9.16. Shearing off a rivet in double cover butt joint.

Let d = Diameter of the rivet hole, τ = Safe permissible shear stress for the rivet material, and n = Number of rivets per pitch length.

We know that shearing area,

$$A_s = \frac{\pi}{4} \times d^2 \quad \text{...(In single shear)}$$

$$= 2 \times \frac{\pi}{4} \times d^2 \quad \text{...(Theoretically, in double shear)}$$

$$= 1.875 \times \frac{\pi}{4} \times d^2 \quad \text{...(In double shear, according to Indian Boiler Regulations)}$$

∴ Shearing resistance or pull required to shear off the rivet per pitch length,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(In single shear)}$$

$$= n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(Theoretically, in double shear)}$$

$$= n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(In double shear, according to Indian Boiler Regulations)}$$

When the shearing resistance (Ps) is greater than the applied load (P) per pitch length, then this type of failure will occur.

4) Crushing of the plate or rivets:

Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. 9.17. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as bearing failure. The area which resists this action is the projected area of the hole or rivet on diametral plane.

The resistance offered by a rivet to be crushed is known as crushing resistance or crushing strength or bearing value of the rivet.

Let d = Diameter of the rivet hole, t = Thickness of the plate, σ_c = Safe permissible crushing stress for the rivet or plate material, and n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (i.e. projected area per rivet),

$$A_c = d \times t$$

∴ Total crushing area = $n \times d \times t$

and crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n \times d \times t \times \sigma_c$$

When the crushing resistance (P_c) is greater than the applied load (P) per pitch length, then this type of failure will occur.

Note: The number of rivets under shear shall be equal to the number of rivets under crushing.

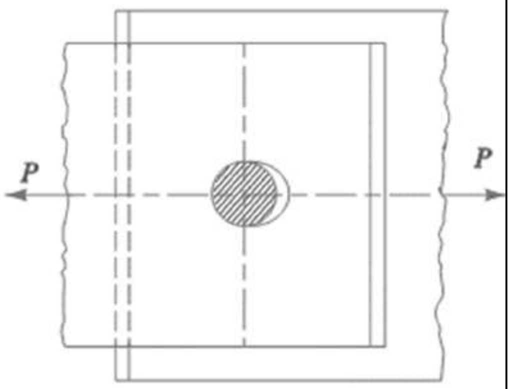


Fig. 9.17. Crushing of a rivet.

Strength of a Riveted Joint:

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail.

We have P_t, P_s and P_c are the pulls required to tear off the plate, shearing off the rivet and crushing off the rivet. A little consideration will show that if we go on increasing the pull on a riveted joint, it will fail when the least of these three pulls is reached, because a higher value of the other pulls will never reach since the joint has failed, either by tearing off the plate, shearing off the rivet or crushing off the rivet.

If the joint is ***continuous*** as in case of boilers, the strength is calculated ***per pitch length***. But if the joint is ***small***, the strength is calculated for the ***whole length*** of the plate.

Efficiency of a Riveted Joint:

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

We have already discussed that strength of the riveted joint

$$= \text{Least of } P_t, P_s \text{ and } P_c$$

Strength of the un-riveted or solid plate per pitch length,

$$P = p \times t \times \sigma_t$$

∴ Efficiency of the riveted joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{p \times t \times \sigma_t}$$

where p = Pitch of the rivets, t = Thickness of the plate, and σ_t = Permissible tensile stress of the plate material.

Example 9.1. *A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint.*

If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution. Given : $t = 15$ mm ; $d = 25$ mm ; $p = 75$ mm ; $\sigma_{tu} = 400$ MPa $= 400$ N/mm^2 ; $\tau_u = 320$ MPa $= 320$ N/mm^2 ; $\sigma_{cu} = 640$ MPa $= 640$ N/mm^2

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)t \times \sigma_{tu} = (75 - 25)15 \times 400 = 300\,000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u = 2 \times \frac{\pi}{4} (25)^2\, 320 = 314\,200 \text{ N} \quad ...(\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480\,000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN. **Ans.**

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300\,000/4 = 75\,000 \text{ N}$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75\,000 = (p - d)\, t \times \sigma_{ta} = (75 - 25)15 \times \sigma_{ta} = 750\, \sigma_{ta}$$

$$\therefore \quad \sigma_{ta} = 75\,000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Actual shearing resistance of the rivets (P_{sa}),

$$75\,000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4} (25)^2\, \tau_a = 982\, \tau_a$$

$$\therefore \quad \tau_a = 75000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \textbf{Ans.}$$

and actual crushing resistance of the rivets (P_{ca}),

$$75\,000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\, \sigma_{ca}$$

$$\therefore \quad \sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Design of Boiler Joints:

The boiler has a longitudinal joint as well as circumferential joint. The **longitudinal joint** is used to join the ends of the plate to get the required diameter of a boiler. For this purpose, a butt joint with two cover plates is used.

The **circumferential joint** is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used.

Assumptions in Designing Boiler Joints:

The following assumptions are made while designing a joint for boilers:

a) The load on the joint is equally shared by all the rivets.
b) The tensile stress is equally distributed over the section of metal between the rivets.
c) The shearing stress in all the rivets is uniform.
d) The crushing stress is uniform.
e) There is no bending stress in the rivets.
f) The holes into which the rivets are driven do not weaken the member.
g) The rivet fills the hole after it is driven.
h) The friction between the surfaces of the plate is neglected.

Design of Longitudinal Butt Joint for a Boiler:

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1) Thickness of boiler shell:

The thickness of the boiler shell is determined by using the thin cylindrical formula, i.e.

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance}$$

Where, t = Thickness of the boiler shell, P = Steam pressure in boiler, D = Internal diameter of boiler shell, σ_t = Permissible tensile stress, and η_l = Efficiency of the longitudinal joint.

The following points may be noted:

(a) The thickness of the boiler shell should not be less than 7 mm.

(b) The efficiency of the joint may be taken from the table

Table 9.1. Efficiencies of commercial boiler joints.

Lap joints	*Efficiency (%)*	**Maximum efficiency*	*Butt joints (Double strap)*	*Efficiency (%)*	**Maximum efficiency*
Single riveted	45 to 60	63.3	Single riveted	55 to 60	63.3
Double riveted	63 to 70	77.5	Double riveted	70 to 83	86.6
Triple riveted	72 to 80	86.6	Triple riveted (5 rivets per pitch with unequal width of straps)	80 to 90	95.0
			Quadruple riveted	85 to 94	98.1

* The maximum efficiencies are valid for ideal equistrength joints with tensile stress = 77 MPa, shear stress = 62 MPa and crushing stress = 133 MPa.

Indian Boiler Regulations (I.B.R.) allows a maximum efficiency of 85% for the best joint.

(c) According to I.B.R., the factor of safety should not be less than 4 and can be taken from the table.

Table 9.2. Factor of safety for boiler joints.

Type of joint	Factor of safety	
	Hand riveting	Machine riveting
Lap joint	4.75	4.5
Single strap butt joint	4.75	4.5
Single riveted butt joint with two equal cover straps	4.75	4.5
Double riveted butt joint with two equal cover straps	4.25	4.0

2) Diameter of rivets:

After finding out the thickness of the boiler shell (t), the diameter of the rivet hole (d) may be determined by using Unwin's empirical formula, i.e.

$$d = 6\sqrt{t} \quad \text{(when t is greater than 8 mm)}$$

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing.

Table 9.3. Size of rivet diameters for rivet hole diameter as per IS : 1928 – 1961 (Reaffirmed 1996).

Basic size of rivet mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48
Rivet hole diameter (min) mm	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44	50

3) Pitch of rivets:

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that

a) The pitch of the rivets should not be less than 2d, which is necessary for the formation of head.

b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is.

$$p_{max} = C \times t + 41.28 \text{ mm}$$

where t = Thickness of the shell plate in mm, and C = Constant.

Table 9.4. Preferred length and diameter combinations for rivets used in boilers as per IS : 1928–1961 (Reaffirmed 1996).

(All dimensions in mm)

Length	*Diameter*													
	12	14	16	18	20	22	24	27	30	33	36	39	42	48
28	×	–	–	–	–	–	–	–	–	–	–	–	–	–
31.5	×	×	–	–	–	–	–	–	–	–	–	–	–	–
35.5	×	×	×	–	–	–	–	–	–	–	–	–	–	–
40	×	×	×	×	–	–	–	–	–	–	–	–	–	–
45	×	×	×	×	×	–	–	–	–	–	–	–	–	–
50	×	×	×	×	×	×	–	–	–	–	–	–	–	–
56	×	×	×	×	×	×	×	–	–	–	–	–	–	–
63	×	×	×	×	×	×	×	×	–	–	–	–	–	–
71	×	×	×	×	×	×	×	×	×	–	–	–	–	–
80	×	×	×	×	×	×	×	×	×	–	–	–	–	–
85	–	×	×	×	×	×	×	×	×	×	–	–	–	–
90	–	×	×	×	×	×	×	×	×	×	–	–	–	–
95	–	×	×	×	×	×	×	×	×	×	×	–	–	–
100	–	–	×	×	×	×	×	×	×	×	×	–	–	–
106	–	–	×	×	×	×	×	×	×	×	×	×	–	–
112	–	–	×	×	×	×	×	×	×	×	×	×	–	–
118	–	–	–	×	×	×	×	×	×	×	×	×	×	–
125	–	–	–	–	×	×	×	×	×	×	×	×	×	×
132	–	–	–	–	–	×	×	×	×	×	×	×	×	×
140	–	–	–	–	–	×	×	×	×	×	×	×	×	×
150	–	–	–	–	–	–	×	×	×	×	×	×	×	×
160	–	–	–	–	–	–	×	×	×	×	×	×	×	×
180	–	–	–	–	–	–	–	×	×	×	×	×	×	×
200	–	–	–	–	–	–	–	–	×	×	×	×	×	×
224	–	–	–	–	–	–	–	–	–	×	×	×	×	×
250	–	–	–	–	–	–	–	–	–	–	–	–	×	×

Preferred numbers are indicated by ×.

Table 9.5. Values of constant *C*.

Number of rivets per pitch length	*Lap joint*	*Butt joint (single strap)*	*Butt joint (double strap)*
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

Note: If the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than p_{max}, then the value of p_{max} is taken.

4) Distance between the rows of rivets:

The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows:

a) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets (p_b) should not be less than

$0.33\, p + 0.67\, d$, for zig-zig riveting, and

2d, for chain riveting.

b) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than

$0.33\, p + 0.67$ or 2d, whichever is greater.

The distance between the rows in which there are full numbers of rivets shall not be less than 2d.

c) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than

$0.2\, p + 1.15\, d$.

The distance between the rows in which there are full numbers of rivets (zig-zag) shall not be less than $0.165\, p + 0.67\, d$.

Note: In the above discussion, p is the pitch of the rivets in the outer rows..

5) Thickness of butt strap:

According to I.B.R., the thicknesses for butt strap (t_1) are as given below:

a) The thickness of butt strap, in no case, shall be less than 10 mm.

b) t_1=1.125 t, for ordinary (chain riveting) single butt strap.

$t_1 = 1.125\, t \left(\dfrac{p-d}{p-2d}\right)$, for single butt straps, every alternate rivet in outer rows being omitted.

$t_1 = 0.625\, t$, for double butt-straps of equal width having ordinary riveting (chain riveting).

$t_1 = 0.625\, t \left(\dfrac{p-d}{p-2d}\right)$, for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

c) For unequal width of butt straps, the thicknesses of butt strap are

$t_1 = 0.75\, t$, for wide strap on the inside, and

$t_2 = 0.625\, t$, for narrow strap on the outside.

6) Margin:

The margin (m) is taken as 1.5 d.

Design of Circumferential Lap Joint for a Boiler:

1) Thickness of the shell and diameter of rivets:

The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

2) Number of rivets:

Since it is a lap joint, therefore the rivets will be in single shear.

∴ Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$

where n = Total number of rivets.

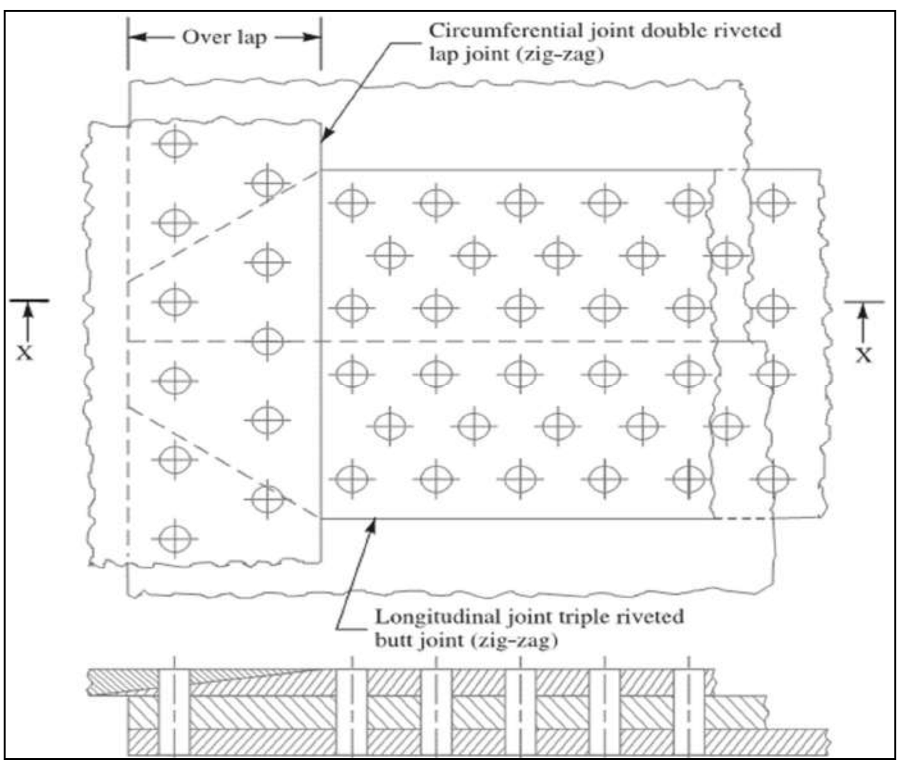
Knowing the inner diameter of the boiler shell (D), and the pressure of steam (P), the total shearing load acting on the circumferential joint,

$$W_s = \frac{\pi}{4} \times D^2 \times P$$

From equations (i) and (ii), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\therefore \qquad n = \left(\frac{D}{d}\right)^2 \frac{P}{\tau}$$



Longitudinal and circumferential joint

3) Pitch of rivets:

If the efficiency of the longitudinal joint is known, then the efficiency of the circumferential joint may be obtained. It is generally taken as 50% of tearing efficiency in longitudinal joint, but if more than one circumferential joint is used, then it is 62% for the intermediate joints. Knowing the efficiency of the circumferential lap joint (η_c), the pitch of the rivets for the lap joint (p_1) may be obtained by using the relation:

$$\eta_c = \frac{p_1 - d}{p_1}$$

4) Number of rows:

The number of rows of rivets for the circumferential joint may be obtained from the following relation:

$$\text{Number of rows} = \frac{\text{Total number of rivets}}{\text{Number of rivets in one row}}$$

and the number of rivets in one row

$$= \frac{\pi (D + t)}{p_1}$$

where D = Inner diameter of shell.

5) After finding out the number of rows, the type of the joint (i.e. single riveted or double riveted etc.) may be decided. Then the number of rivets in a row and pitch may be re-adjusted. In order to have a leak-proof joint, the pitch for the joint should be checked from Indian Boiler Regulations.

6) The distance between the rows of rivets (i.e. back pitch) is calculated by using the relations as discussed in the previous article.

7) After knowing the distance between the rows of rivets (p_b), the overlap of the plate may be fixed by using the relation,

$$\text{Overlap} = (\text{No. of rows of rivets} - 1)\, p_b + m$$

where m = Margin.

Table 9.6. Recommended joints for pressure vessels.

Diameter of shell (metres)	*Thickness of shell (mm)*	*Type of joint*
0.6 to 1.8	6 to 13	Double riveted
0.9 to 2.1	13 to 25	Triple riveted
1.5 to 2.7	19 to 40	Quadruple riveted

Example 9.10. *A steam boiler is to be designed for a working pressure of 2.5 N/mm² with its inside diameter 1.6 m. Give the design calculations for the longitudinal and circumferential joints for the following working stresses for steel plates and rivets :*

In tension = 75 MPa ; In shear = 60 MPa; In crushing = 125 MPa.

Draw the joints to a suitable scale.

Solution. Given : $P = 2.5$ N/mm² ; $D = 1.6$ m $= 1600$ mm ; $\sigma_t = 75$ MPa $= 75$ N/mm² ; $\tau = 60$ MPa $= 60$ N/mm² ; $\sigma_c = 125$ MPa $= 125$ N/mm²

Design of longitudinal joint

The longitudinal joint for a steam boiler may be designed as follows :

1. *Thickness of boiler shell*

We know that the thickness of boiler shell,

$$t = \frac{P.D}{2\,\sigma_t} + 1 \text{ mm} = \frac{2.5 \times 1600}{2 \times 75} + 1 \text{ mm}$$

$$= 27.6 \text{ say } 28 \text{ mm} \quad \textbf{Ans.}$$

2. *Diameter of rivet*

Since the thickness of the plate is more than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{28} = 31.75 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 34.5 mm and the corresponding diameter of the rivet is 33 mm. **Ans.**

3. *Pitch of rivets*

Assume the joint to be triple riveted double strap butt joint with unequal cover straps, as shown in Fig. 9.11.

Let p = Pitch of the rivet in the outer most row.

∴ Tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (p - 34.5)\, 28 \times 75 \text{ N}$$

$$= 2100\,(p - 34.5) \text{ N} \qquad \ldots(i)$$

Since the joint is triple riveted with two unequal cover straps, therefore there are 5 rivets per pitch length. Out of these five rivets, four are in double shear and one is in single shear. Assuming the strength of rivets in double shear as 1.875 times that of single shear, therefore

Shearing resistance of the rivets per pitch length,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4}\,(34.5)^2\, 60 = 476\,820 \text{ N} \qquad \ldots(ii)$$

Equating equations (i) and (ii), we get

$$2100\,(p - 34.5) = 476\,820$$

$$\therefore \quad p - 34.5 = 476\,820 / 2100 = 227 \text{ or } p = 227 + 34.5 = 261.5 \text{ mm}$$

According to I.B.R., the maximum pitch,

$$p_{max} = C.t + 41.28 \text{ mm}$$

From Table 9.5, we find that for double strap butt joint with 5 rivets per pitch length, the value of C is 6.

$$\therefore \quad p_{max} = 6 \times 28 + 41.28 = 209.28 \text{ say } 220 \text{ mm}$$

Since p_{max} is less than p, therefore we shall adopt

$$p = p_{max} = 220 \text{ mm} \quad \textbf{Ans.}$$

∴ Pitch of rivets in the inner row,

$$p' = 220 / 2 = 110 \text{ mm} \quad \textbf{Ans.}$$

4. *Distance between the rows of rivets*

According to I.B.R., the distance between the outer row and the next row

$$= 0.2\,p + 1.15\,d = 0.2 \times 220 + 1.15 \times 34.5 \text{ mm}$$

$$= 83.7 \text{ say } 85 \text{ mm} \quad \textbf{Ans.}$$

and the distance between the inner rows for zig-zig riveting

$$= 0.165\, p + 0.67\, d = 0.165 \times 220 + 0.67 \times 34.5 \text{ mm}$$
$$= 59.4 \text{ say } 60 \text{ mm} \quad \textbf{Ans.}$$

5. *Thickness of butt straps*

We know that for unequal width of butt straps, the thicknesses are :

For wide butt strap, $t_1 = 0.75\, t = 0.75 \times 28 = 21$ mm **Ans.**

and for narrow butt strap, $t_2 = 0.625\, t = 0.625 \times 28 = 17.5$ say 18 mm **Ans.**

It may be noted that the wide and narrow butt straps are placed on the inside and outside of the shell respectively.

6. *Margin*

We know that the margin,

$$m = 1.5\, d = 1.5 \times 34.5 = 51.75 \text{ say } 52 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the efficiency of the designed joint.

Tearing resistance of the plate in the outer row,

$$P_t = (p - d)\, t \times \sigma_t = (220 - 34.5)\, 28 \times 75 = 389\,550 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau$$
$$= 8.5 \times \frac{\pi}{4} (34.5)^2\, 60 = 476\,820 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 5 \times 34.5 \times 28 \times 125 = 603\,750 \text{ N}$$

The joint may also fail by tearing off the plate between the rivets in the second row. This is only possible if the rivets in the outermost row gives way (*i.e.* shears). Since there are two rivet holes per pitch length in the second row and one rivet in the outermost row, therefore

Combined tearing and shearing resistance

$$= (p - 2d)\, t \times \sigma_t + \frac{\pi}{4} \times d^2 \times \tau$$
$$= (220 - 2 \times 34.5)\, 28 \times 75 + \frac{\pi}{4} (34.5)^2\, 60$$
$$= 317\,100 + 56\,096 = 373\,196 \text{ N}$$

From above, we see that the strength of the joint

$$= 373\,196 \text{ N}$$

Strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 220 \times 28 \times 75 = 462\,000 \text{ N}$$

$\therefore$ Efficiency of the designed joint,

$$\eta = \frac{373\,196}{462\,000} = 0.808 \text{ or } 80.8\% \quad \textbf{Ans.}$$

Design of circumferential joint

The circumferential joint for a steam boiler may be designed as follows :

1. The thickness of the boiler shell (t) and diameter of rivet hole (d) will be same as for longitudinal joint, *i.e.*

$$t = 28 \text{ mm} \text{ ; and } d = 34.5 \text{ mm}$$

2. *Number of rivets*

Let n = Number of rivets.

We know that shearing resistance of the rivets

$$= n \times \frac{\pi}{4} \times d^2 \times \tau \quad ...(i)$$

and total shearing load acting on the circumferential joint

$$= \frac{\pi}{4} \times D^2 \times P \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\therefore \quad n = \frac{D^2 \times P}{d^2 \times \tau} = \frac{(1600)^2 \; 2.5}{(34.5)^2 \; 60} = 89.6 \text{ say } 90 \quad \textbf{Ans.}$$

3. *Pitch of rivets*

Assuming the joint to be double riveted lap joint with zig-zag riveting, therefore number of rivets per row

$$= 90 / 2 = 45$$

We know that the pitch of the rivets,

$$p_1 = \frac{\pi (D + t)}{\text{Number of rivets per row}} = \frac{\pi (1600 + 28)}{45} = 113.7 \text{ mm}$$

Let us take pitch of the rivets, p_1 = 140 mm **Ans.**

4. *Efficiency of the joint*

We know that the efficiency of the circumferential joint,

$$\eta_c = \frac{p_1 - d}{p_1} = \frac{140 - 34.5}{140} = 0.753 \text{ or } 75.3\%$$

5. *Distance between the rows of rivets*

We know that the distance between the rows of rivets for zig-zag riveting,

$$= 0.33 \, p_1 + 0.67 \, d = 0.33 \times 140 + 0.67 \times 34.5 \text{ mm}$$
$$= 69.3 \text{ say } 70 \text{ mm} \quad \textbf{Ans.}$$

6. *Margin*

We know that the margin,

$$m = 1.5 \, d = 1.5 \times 34.5$$
$$= 51.75 \text{ say } 52 \text{ mm} \quad \textbf{Ans.}$$

Example 9.9. *Design the longitudinal joint for a 1.25 m diameter steam boiler to carry a steam pressure of 2.5 N/mm². The ultimate strength of the boiler plate may be assumed as 420 MPa, crushing strength as 650 MPa and shear strength as 300 MPa. Take the joint efficiency as 80%. Sketch the joint with all the dimensions. Adopt the suitable factor of safety.*

Solution. Given : D = 1.25 m = 1250 mm; P = 2.5 N/mm² ; σ_{tu} = 420 MPa = 420 N/mm²; σ_{cu} = 650 MPa = 650 N/mm² ; τ_u = 300 MPa = 300 N/mm² ; η_l = 80% = 0.8

Assuming a factor of safety (*F.S.*) as 5, the allowable stresses are as follows :

$$\sigma_t = \frac{\sigma_{tu}}{F.S.} = \frac{420}{5} = 84 \text{ N/mm}^2$$

$$\sigma_c = \frac{\sigma_{cu}}{F.S.} = \frac{650}{5} = 130 \text{ N/mm}^2$$

and $$\tau = \frac{\tau_u}{F.S.} = \frac{300}{5} = 60 \text{ N/mm}^2$$

1. *Thickness of plate*

We know that thickness of plate,

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{2.5 \times 1250}{2 \times 84 \times 0.8} + 1 \text{ mm}$$
$$= 24.3 \text{ say } 25 \text{ mm} \quad \textbf{Ans.}$$

2. *Diameter of rivet*

Since the thickness of the plate is more than 8 mm, therefore diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{25} = 30 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole is 31.5 mm and the corresponding diameter of the rivet is 30 mm. **Ans.**

3. *Pitch of rivets*

Assume a triple riveted double strap butt joint with unequal straps, as shown in Fig. 9.11.

Let p = Pitch of the rivets in the outer most row.

∴ Tearing strength of the plate per pitch length,

$$P_t = (p - d)\,t \times \sigma_t = (p - 31.5)\,25 \times 84 = 2100\,(p - 31.5)\text{ N} \quad ...(i)$$

Since the joint is triple riveted with two unequal cover straps, therefore there are 5 rivets per pitch length. Out of these five rivets, four rivets are in double shear and one is in single shear. Assuming the strength of the rivets in double shear as 1.875 times that of single shear, therefore

Shearing resistance of the rivets per pitch length,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau$$
$$= 8.5 \times \frac{\pi}{4}\,(31.5)^2\,60 = 397\,500 \text{ N} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$2100\,(p - 31.5) = 397\,500$$

$$\therefore \quad p - 31.5 = 397\,500 / 2100 = 189.3 \text{ or } p = 31.5 + 189.3 = 220.8 \text{ mm}$$

According to I.B.R., maximum pitch,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for double strap butt joint with 5 rivets per pitch length, the value of C is 6.

$$\therefore \quad p_{max} = 6 \times 25 + 41.28 = 191.28 \text{ say } 196 \text{ mm} \quad \textbf{Ans.}$$

Since p_{max} is less than p, therefore we shall adopt $p = p_{max}$ = 196 mm **Ans.**

∴ Pitch of rivets in the inner row,

$$p' = 196 / 2 = 98 \text{ mm} \quad \textbf{Ans.}$$

4. *Distance between the rows of rivets*

According to I.B.R., the distance between the outer row and the next row,

$$= 0.2\,p + 1.15\,d = 0.2 \times 196 + 1.15 \times 31.5 \text{ mm}$$
$$= 75.4 \text{ say } 76 \text{ mm} \quad \textbf{Ans.}$$

and the distance between the inner rows for zig-zag riveting

$$= 0.165\,p + 0.67\,d = 0.165 \times 196 + 0.67 \times 31.5 \text{ mm}$$
$$= 53.4 \text{ say } 54 \text{ mm} \quad \textbf{Ans.}$$

5. *Thickness of butt straps*

We know that for unequal width of butt straps, the thicknesses are as follows :

For wide butt strap, $t_1 = 0.75\, t = 0.75 \times 25 = 18.75$ say 20 mm Ans.

and for narrow butt strap, $t_2 = 0.625\, t = 0.625 \times 25 = 15.6$ say 16 mm Ans.

It may be noted that wide and narrow butt straps are placed on the inside and outside of the shell respectively.

6. *Margin*

We know that the margin,

$$m = 1.5\, d = 1.5 \times 31.5 = 47.25 \text{ say } 47.5 \text{ mm} \quad \text{Ans.}$$

Let us now check the efficiency of the designed joint.

Tearing resistance of the plate in the outer row,

$$P_t = (p - d)\, t \times \sigma_t = (196 - 31.5)\, 25 \times 84 = 345\,450 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4} (31.5)^2 \times 60 = 397\,500 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 5 \times 31.5 \times 25 \times 130 = 511\,875 \text{ N} \quad ...(\because n = 5)$$

The joint may also fail by tearing off the plate between the rivets in the second row. This is only possible if the rivets in the outermost row gives way (*i.e.* shears). Since there are two rivet holes per pitch length in the second row and one rivet is in the outer most row, therefore combined tearing and shearing resistance

$$= (p - 2d)\, t \times \sigma_t + \frac{\pi}{4} \times d^2 \times \tau$$

$$= (196 - 2 \times 31.5)\, 25 \times 84 + \frac{\pi}{4} (31.5)^2\, 60 = 326\,065 \text{ N}$$

From above, we see that strength of the joint

$$= 326\,065 \text{ N}$$

Strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 196 \times 25 \times 84 = 411\,600 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = 326\,065 / 411\,600 = 0.792 \text{ or } 79.2\%$$

Since the efficiency of the designed joint is nearly equal to the given efficiency, therefore the design is satisfactory.

Example 9.7. *Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm². Assume joint efficiency as 75%, allowable tensile stress in the plate 90 MPa ; compressive stress 140 MPa ; and shear stress in the rivet 56 MPa.*

Solution. Given : $D = 1.5$ m $= 1500$ mm ; $P = 0.95$ N/mm^2 ; $\eta_l = 75\% = 0.75$; $\sigma_t = 90$ MPa $= 90$ N/mm^2 ; $\sigma_c = 140$ MPa $= 140$ N/mm^2 ; $\tau = 56$ MPa $= 56$ N/mm^2

1. *Thickness of boiler shell plate*

We know that thickness of boiler shell plate,

$$t = \frac{P.D}{2\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12 \text{ mm} \quad \text{Ans.}$$

2. *Diameter of rivet*

Since the thickness of the plate is greater than 8 mm, therefore the diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.8 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 21 mm and the corresponding diameter of the rivet is 20 mm. Ans.

3. *Pitch of rivets*

Let p = Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 21)12 \times 90 = 1080\,(p - 21)\text{N} \quad \text{...}(i)$$

Since the joint is double riveted double strap butt joint, as shown in Fig. 9.9, therefore there are two rivets per pitch length (*i.e.* $n = 2$) and the rivets are in double shear. Assuming that the rivets in double shear are 1.875 times stronger than in single shear, we have

Shearing strength of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4}\,(21)^2 \times 56 \text{ N}$$

$$= 72\,745 \text{ N} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we get

$$1080\,(p - 21) = 72\,745$$

$$\therefore \quad p - 21 = 72\,745 / 1080 = 67.35 \text{ or } p = 67.35 + 21 = 88.35 \text{ say } 90 \text{ mm}$$

According to I.B.R., the maximum pitch of rivets for longitudinal joint of a boiler is given by

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for a double riveted double strap butt joint and two rivets per pitch length, the value of C is 3.50.

$$\therefore \quad p_{max} = 3.5 \times 12 + 41.28 = 83.28 \text{ say } 84 \text{ mm}$$

Since the value of p is more than p_{max}, therefore we shall adopt pitch of the rivets,

$$p = p_{max} = 84 \text{ mm} \quad \textbf{Ans.}$$

4. *Distance between rows of rivets*

Assuming zig-zag riveting, the distance between the rows of the rivets (according to I.B.R.),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 84 + 0.67 \times 21 = 41.8 \text{ say } 42 \text{ mm} \quad \textbf{Ans.}$$

5. *Thickness of cover plates*

According to I.B.R., the thickness of each cover plate of equal width is

$$t_1 = 0.625\,t = 0.625 \times 12 = 7.5 \text{ mm} \quad \textbf{Ans.}$$

6. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 21 = 31.5 \text{ say } 32 \text{ mm} \quad \textbf{Ans.}$$

Let us now find the efficiency for the designed joint.

Tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (84 - 21)12 \times 90 = 68\,040 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4}\,(21)^2 \times 56 = 72\,745 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = 70\,560 \text{ N}$$

Since the strength of riveted joint is the least value of P_t, P_s or P_c, therefore strength of the riveted joint,

$$P_t = 68\,040 \text{ N}$$

We know that strength of the un-riveted plate,

$$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90\,720 \text{ N}$$

$\therefore$ Efficiency of the designed joint,

$$\eta = \frac{P_t}{P} = \frac{68\,040}{90\,720} = 0.75 \text{ or } 75\% \quad \textbf{Ans.}$$

Since the efficiency of the designed joint is equal to the given efficiency of 75%, therefore the design is satisfactory.

Chapter 03: Design of Shafts and Keys

Shafts and its function:

- A shaft is a rotating machine element which is used to transmit power from one place to another.
- The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.
- In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it.
- These members along with the forces exerted upon them causes the shaft to bending.
- In other words, a shaft is used for the transmission of torque and bending moment.

Material for Shafts:

The material used for shafts should have the following properties:

- It should have high strength.
- Good machinability.
- Low notch sensitivity factor.
- Good heat treatment properties.
- High wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

Stresses in Shafts:

a) Shear stresses due to the transmission of torque (i.e. due to torsional load).
b) Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
c) Stresses due to combined torsional and bending loads.

Maximum Permissible Working Stresses for Transmission Shafts:

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as.

a) 112 MPa for shafts without allowance for keyways.
b) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

$$\sigma_t = 0.6\ \sigma_{el} \text{ or } 0.36\ \sigma_u, \text{ whichever is less.}$$

The maximum permissible shear stress may be taken as

a) 56 MPa for shafts without allowance for key ways.
b) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σ_{el} but not more than 18 per cent of the ultimate tensile strength (σ_u). In other words, the permissible shear stress,

$$\tau = 0.3\ \sigma_{el} \text{ or } 0.18\ \sigma_u, \text{ whichever is less.}$$

Design of Shafts on the basis of Strength:

a) Shafts subjected to twisting moment or torque only,
b) Shafts subjected to bending moment only,
c) Shafts subjected to combined twisting and bending moments, and
d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

a) Shafts subjected to Twisting Moment or Torque only:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that.

$$\frac{T}{J} = \frac{\tau}{r} \qquad \ldots(i)$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= d/2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (i) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \ldots(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i= Outside and inside diameter of the shaft, and r = d_o /2.

Substituting these values in equation (i), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \ldots(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft= d_i/d_o

Now the equation (iii) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \qquad \ldots(iv)$$

Notes:

a) The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

b) The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

Where T = Twisting moment in N-m, and N = Speed of the shaft in r.p.m.

c) In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : $N = 200$ r.p.m. ; $P = 20$ kW $= 20 \times 10^3$ W; $\tau = 42$ MPa $= 42$ N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.25 = 115\ 733$ or $d = 48.7$ say 50 mm **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\, T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

$\therefore$ Maximum torque transmitted,

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\,d^3$$

$$\therefore \quad d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

or $$d = 159.4 \text{ say } 160 \text{ mm Ans.}$$

Example 14.3. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.*

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; $F.S. = 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\,d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm Ans.}$$

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\,(d_o)^3\,[1 - (0.5)^4] = 8.3\,(d_o)^3$$

$$\therefore \quad (d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm Ans.}$$

and $$d_i = 0.5\,d_o = 0.5 \times 50 = 25 \text{ mm Ans.}$$

b) Shafts subjected to Bending Moment only:

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that.

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \ldots(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \text{...(where } k = d_i / d_o)$$

$$y = d_o / 2$$

Again substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Example 14.4. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Solution. Given : $W = 50$ kN $= 50 \times 10^3$ N ; $L = 100$ mm ; $x = 1.4$ m ; $\sigma_b = 100$ MPa $= 100$ N/mm^2

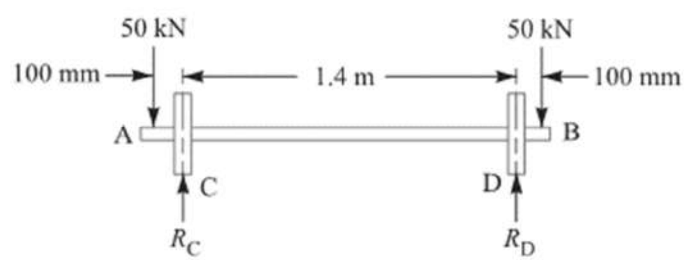


Fig. 14.1

The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$*M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm Ans.}$$

c) Shafts subjected to Combined Twisting Moment and Bending Moment:

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. The following two theories are important from the subject point of view:

a) Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
b) Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b=Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b, we have,

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad \ldots(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (i) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \ldots(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment. By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (iv) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Note: 1. In case of a hollow shaft, the equations **(ii)** and **(v)** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

and $$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Solution. Given : M = 3000 N-m = 3×10^6 N-mm ; T = 10 000 N-m = 10×10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\ d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm Ans.

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : P = 100 kW = 100×10^3 W ; N = 300 r.p.m. ; L = 3 m ; W = 1500 N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

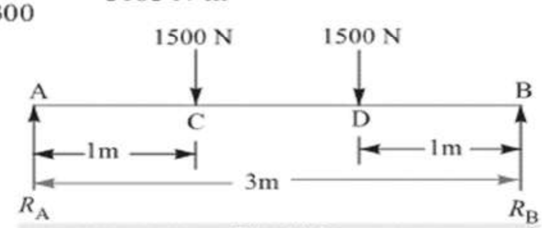


Fig. 14.3

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at C and D.

$\therefore$ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$
$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

$$\therefore \quad d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3 \text{ or } d = 66.8 \text{ say } 70 \text{ mm } \textbf{Ans.}$$

d) Shafts subjected Axial Load in addition to Combined Torsion and Bending Loads:

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is,

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \quad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \quad \text{... } (\because k = d_i/d_o)$$

$\therefore$ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32\,M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \quad \text{...}(i)$$

$$= \frac{32\,M_1}{\pi d^3} \quad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32\,M_1}{\pi(d_o)^3\,(1 - k^4)}$$

$$\text{...}\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1 + k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

$\therefore$ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 (1 - k^2)} \quad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation:

$$\text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044 (L/K)}$$

where M = Bending moment,

$$\text{**Column factor,} \quad \alpha = \frac{\sigma_y (L/K)^2}{C \pi^2 E}$$

Where L = Length of shaft between the bearings,

K = Least radius of gyration

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

and

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

It may be noted that for a solid shaft, k = 0 and d_0 = d. When the shaft carries no axial load, then F = 0 and when the shaft carries axial tensile load, then α = 1.

Example 14.18. *A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.*

Solution. Given : $T = 1.5$ kN-m $= 1.5 \times 10^3$ N-m ; $M = 3$ kN-m $= 3 \times 10^3$ N-m ; $F = 10$ kN $= 10 \times 10^3$ N ; $k = d_i / d_o = 0.5$; $d_o = 80$ mm $= 0.08$ m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from Table 14.2, we find that

$K_m = 1.5$; and $K_t = 1.0$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1 + 0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

... ($\because$ $\alpha = 1$, for axial tensile loading)

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) = \frac{\pi}{16} \times \tau (80)^3 (1 - 0.5^4) = 94\,260\ \tau$$

$\therefore$ $\tau = 4862 \times 10^3 / 94\,260 = 51.6$ N/mm^2 $= 51.6$ MPa **Ans.**

Example 14.19. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. *The maximum shear stress developed in the shaft, and*
2. *The angular twist between the bearings.*

Solution. Given : $d_o = 0.5$ m ; $d_i = 0.3$ m ; $P = 5600$ kW $= 5600 \times 10^3$ W ; $L = 6$ m ; $N = 150$ r.p.m. ; $F = 500$ kN $= 500 \times 10^3$ N ; $W = 70$ kN $= 70 \times 10^3$ N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\,460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\,500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L/K = 6/0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $\qquad k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\,500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\,(1 + 0.6^2)}{8}\right]^2 + (1 \times 356\,460)^2}$$

$$= \sqrt{(78\,750 + 51\,850)^2 + (356\,460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\,[1 - (0.6)^4] = 0.02\,\tau$$

$$\therefore \qquad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19 \text{ MPa } \textbf{Ans.}$$

2. *Angular twist between the bearings*

Let $\qquad \theta$ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}\,[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

Design of Shafts on the basis of Rigidity:

a) Torsional Rigidity:

The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3° per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1° in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G \cdot \theta}{L} \quad \text{or} \quad \theta = \frac{T \cdot L}{J \cdot G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \qquad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft

b) Lateral Rigidity:

It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, i.e.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa = 84×10^9 N/m² = 84×10^3 N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or

$$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$$\therefore \quad d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6 \quad \text{or} \quad d = 33.87 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$$\therefore \quad \tau = 47\,740 / 8420 = 5.67 \text{ N/mm}^2 = 5.67 \text{ MPa } \textbf{Ans.}$$

Types of Shaft:

The following two types of shafts are important from the subject point of view:

1. Transmission shafts. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. Machine shafts. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Size of Shaft as per I.S.:

The standard sizes of transmission shafts are:

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.

The standard lengths of the shafts are 5 m, 6 m and 7 m.

Keys

Keys and its function:

- A key is a machine element used to connect a rotating machine element to a shaft.
- The key prevents relative rotation between the two parts and may enable torque transmission.
- For a key to function, the shaft and rotating machine element must have a keyway and a keyseat, which is a slot and pocket in which the key fits. The whole system is called a keyed joint.
- It is always inserted parallel to the axis of the shaft.
- A keyed joint may allow relative axial movement between the parts.
- Commonly keyed components include gears, pulleys, couplings, and washers.

Types of Keys:

The material used for shafts should have the following properties:

1. Sunk keys,
2. Saddle keys,
3. Tangent keys,
4. Round keys
5. Splines

1) Sunk keys:

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.

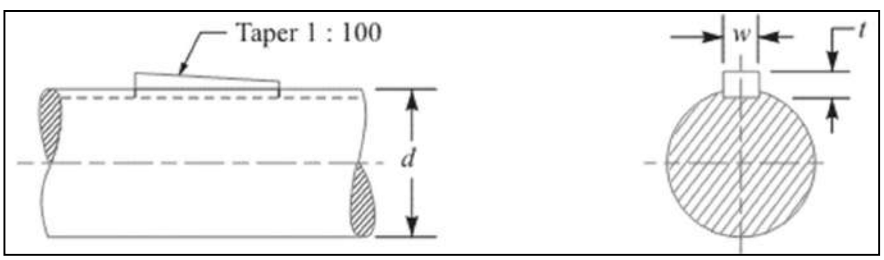
a) Rectangular sunk key:

The usual proportions of this key are:

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

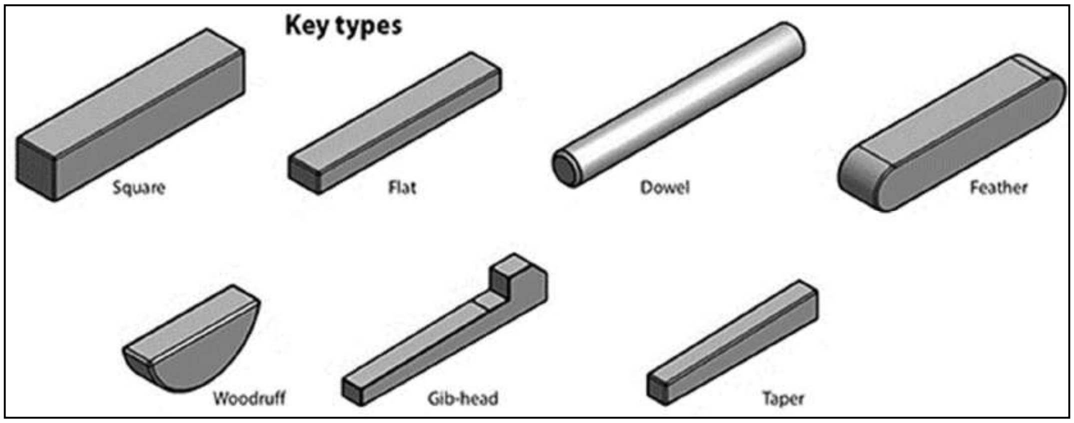
where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only..



b) Square sunk key:

The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal.

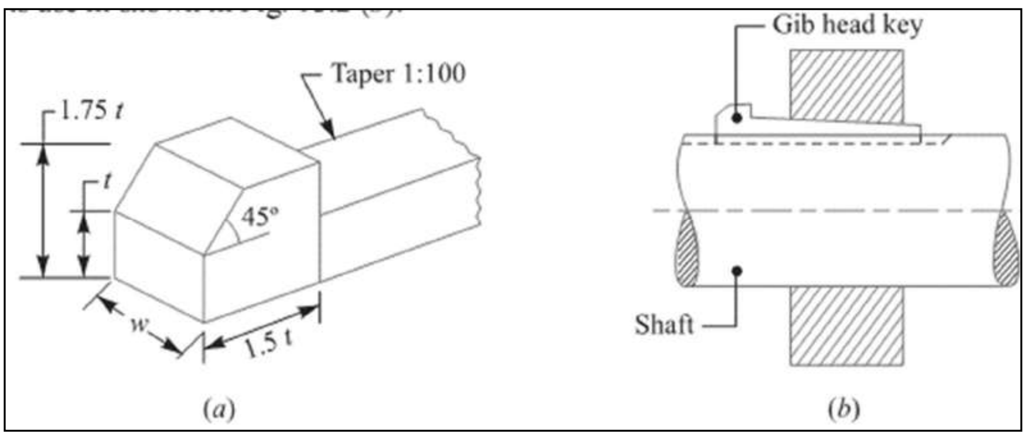


c) Parallel sunk key:

The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

d) Gib-head key:

It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key.



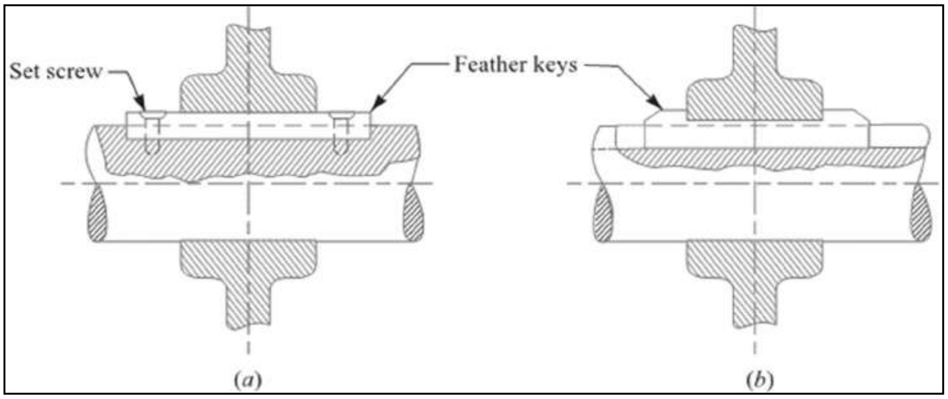
The usual proportions of the gib head key are:

Width, $w = d / 4$;

and thickness at large end, $t = 2w / 3 = d / 6$

e) Feather key:

- A key attached to one member of a pair and which permits relative axial movement is known as feather key.
- It is a special type of parallel key which transmits a turning moment and also permits axial movement.
- It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



- The feather key may be screwed to the shaft or it may have double gib.
- The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

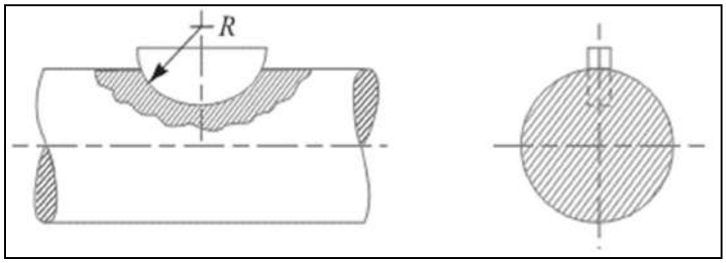
The following table shows the proportions of standard parallel, tapered and gib head keys, according to IS: 2292 and 2293-1974 (Reaffirmed 1992).

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	*Key cross-section*		*Shaft diameter (mm) upto and including*	*Key cross-section*	
	Width (mm)	*Thickness (mm)*		*Width (mm)*	*Thickness (mm)*
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

f) Woodruff key:

- The woodruff key is an easily adjustable key.
- It is a piece from a cylindrical disc having segmental cross-section in front view.
- A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made.
- This key is largely used in machine tool and automobile construction.



Advantages:

- It accommodates itself to any taper in the hub or boss of the mating piece.
- It is useful on tapering shaft ends.
- Its extra depth in the shaft prevents any tendency to turn over in its keyway.

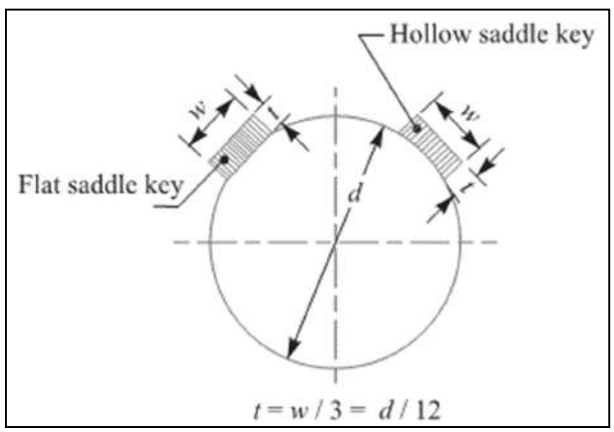
Disadvantages:

- The depth of the keyway weakens the shaft.
- It cannot be used as a feather.

2) Saddle keys:

The saddle keys are of the following two types:

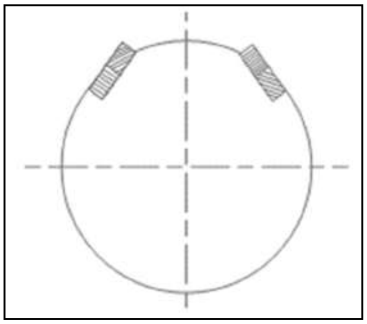
A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.



A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

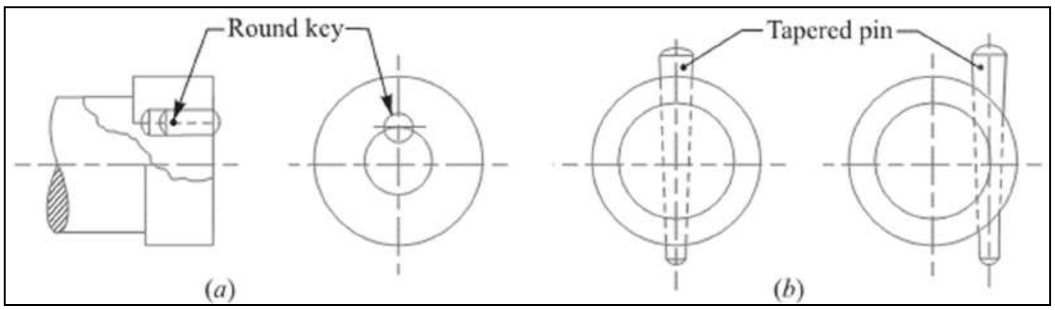
3) Tangent keys:

The tangent keys are fitted in pair at right angles. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.



4) Round keys:

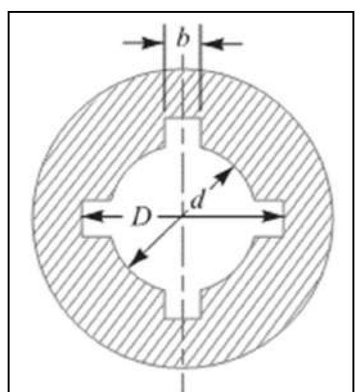
- The round keys are circular in section and fit into holes drilled partly in the shaft and partly in the hub.
- They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled.



- Round keys are usually considered to be most appropriate for low power drives.
- Sometimes the tapered pin is held in place by the friction between the pin and the reamed tapered holes.

5) Splines:

- Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts.
- These shafts usually have four, six, ten or sixteen splines.
- The splined shafts are relatively stronger than shafts having a single keyway.
- The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions.
- By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\ d$ and $b = 0.25\ D$

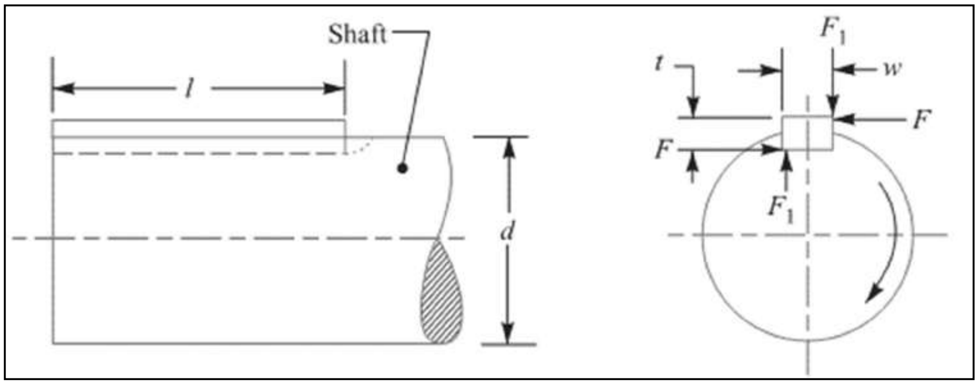
Forces acting on a Sunk Key:

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

1) Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude..

2) Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.



In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

Forces acting on a Sunk Key:

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft, l = Length of key,

w = Width of key, t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad \ldots(i)$$

F = Area resisting crushing × Crushing stress = $l \times \frac{t}{2} \times \sigma_c$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ..[\text{Equating equations } (i) \text{ and } (ii)]$$

or
$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad ...(iii)$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from equation (iii), we have w = t. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad ...(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad ...(v)$$

..(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

∴
$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \qquad ... (\text{Taking } w = d/4) \quad ...(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

∴ $l = 1.571\ d$ ... [From equation (vi)]

Example: *A shaft 30 mm diameter is transmitting power at a maximum shear stress of 80 MPa. If a pulley is connected to the shaft by means of a key, find the dimensions of the key so that the stress in the key is not to exceed 50 MPa and length of the key is 4 times the width.*

Ans:

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : d = 50 mm ; τ = 42 MPa = 42 N/mm^2 ; σ_c = 70 MPa = 70 N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, w = 16 mm **Ans.**

and thickness of key, t = 10 mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \quad ...(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \quad ...(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$$l = 117.7 \text{ say } 120 \text{ mm } \textbf{Ans.}$$

Example 13.2. *A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.*

Solution. Given : d = 45 mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ; t = 9 mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$$\therefore \quad l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\ 213\ l$$

$$\ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$\therefore \quad l = 1.8 \times 10^6 / 17\ 213 = 104.6 \text{ mm}$

Taking the larger of the two values, we have

$l = 104.6$ say 105 mm **Ans.**

Effect of Keyways:

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Example 13.3. *A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 960$ r.p.m. ; $d = 40$ mm ; $l = 75$ mm ; $\tau = 56$ MPa $= 56$ N/mm^2 ; $\sigma_c = 112$ MPa $= 112$ N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\ w$$

$\therefore \quad w = 149 \times 10^3 / 84 \times 10^3 = 1.8 \text{ mm}$

This width of keyway is too small. The width of keyway should be at least $d / 4$.

$$\therefore \qquad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm Ans.}$$

Since $\sigma_c = 2\tau$, therefore a square key of $w = 10$ mm and $t = 10$ mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \qquad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\ (40)^3\ 0.8125 = 571\ 844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\ 000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\ 000}{571\ 844} = 1.47 \text{ Ans.}$$

Chapter 04: Design of Coupling

Coupling:

A coupling is termed as a device used to make permanent or semi-permanent connection where as a clutch permits rapid connection or disconnection at the will of the operator.

Shaft Coupling:

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following:

- To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
- To provide for misalignment of the shafts or to introduce mechanical flexibility.
- To reduce the transmission of shock loads from one shaft to another.
- To introduce protection against overloads.
- It should have no projecting parts.

Requirements of a Good Shaft Coupling:

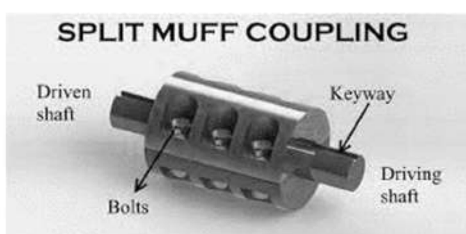
A good shaft coupling should have the following requirements:

- It should be easy to connect or disconnect.
- It should transmit the full power from one shaft to the other shaft without losses.
- It should hold the shafts in perfect alignment.
- It should reduce the transmission of shock loads from one shaft to another shaft.
- It should have no projecting parts.

Types of Shafts Couplings:

a) Rigid coupling: It is used to connect two shafts which are perfectly aligned.

a) Sleeve or muff coupling.

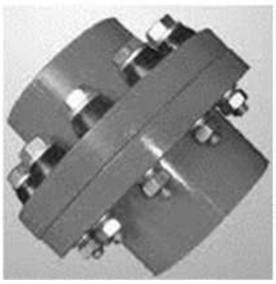


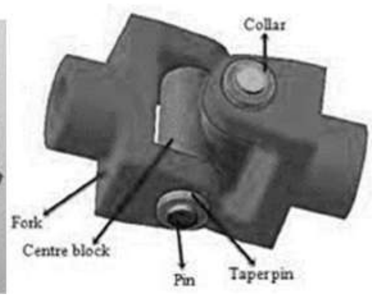
a) Clamp or split-muff or compression coupling, and

b) Flange coupling.

b) Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment.

a) Bushed pin type coupling,
b) Universal coupling, and
c) Oldham coupling.





Sleeve or Muff-coupling:

- It is the simplest type of rigid coupling, made of cast iron.
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.
- It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig.
- The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

Outer diameter of the sleeve, $D = 2d + 13$ mm

and length of the sleeve, $L = 3.5\,d$

where d is the diameter of the shaft

1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft.

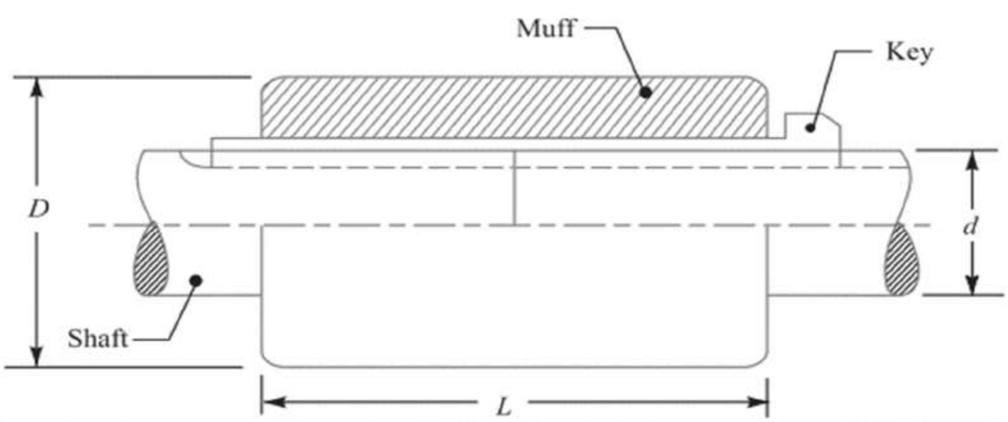


Fig. 13.10. Sleeve or muff coupling.

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \ (1 - k^4) \qquad \ldots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. Design for key

The key for the coupling may be designed in the similar way as discussed in Art. 13.9. The width and thickness of the coupling key is obtained from the proportions.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* 3.5 d). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\, d}{2}$$

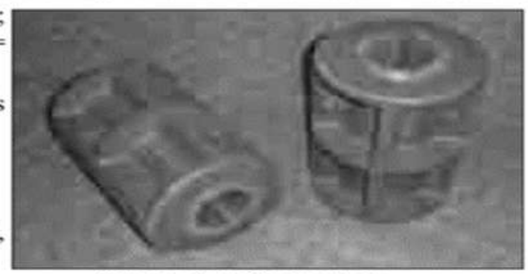
After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \ldots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \ldots \text{(Considering crushing of the key)}$$

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : P = 40 kW = 40 × 10^3 W; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm^2; σ_{cs} = 80 MPa = 80 N/mm^2; τ_c = 15 MPa = 15 N/mm^2



A type of muff couplings.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ d^3 = 1100 × 10^3/7.86 = 140 × 10^3 or d = 52 say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm } \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3\, \tau_c$$

$\therefore$ τ_c = 1100 × 10^3/370 × 10^3 = 2.97 N/mm^2

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm} \textbf{ Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

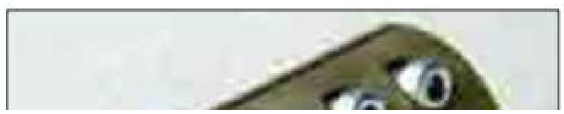
$$\therefore \qquad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3\ \sigma_{cs}$$

$$\therefore \qquad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.



Clamp or Compression Coupling:

- It is also known as split muff coupling. In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig..
- The halves of the muff are made of cast iron. The shaft ends are made to abutt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above.
- Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings.
- This coupling may be used for heavy duty and moderate speeds.
- The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling.

The usual proportions of the muff for the clamp or compression coupling are:

Outer diameter of the muff or sleeve, $D = 2d + 13$ mm

and length of the muff or sleeve, $L = 3.5\ d$

where d is the diameter of the shaft

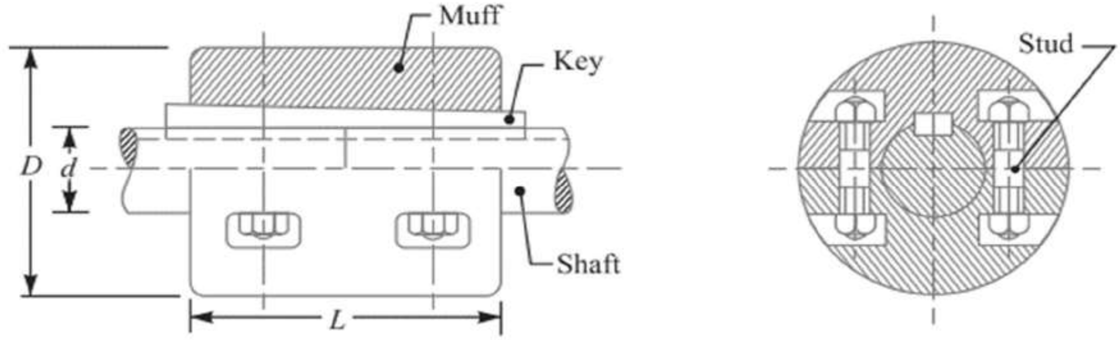


Fig. 13.11. Clamp or compression coupling.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. Design of muff and key

The muff and key are designed in the similar way as discussed in muff coupling.

2. Design of clamping bolts

Let T = Torque transmited by the shaft,
d = Diameter of shaft,
d_b = Root or effective diameter of bolt,
n = Number of bolts,
σ_t = Permissible tensile stress for bolt material,
μ = Coefficient of friction between the muff and shaft, and
L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2\sigma_t$$

$\therefore$ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2\sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2\sigma_t \times \frac{n}{2}}{\frac{1}{2}L \times d}$$

$\therefore$ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2\sigma_t \times \frac{n}{2}}{\frac{1}{2}L \times d} \times \frac{1}{2}\pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note: The value of μ may be taken as 0.3.

Example 13.5. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm **Ans.**

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm} \textbf{ Ans.}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm} \textbf{ Ans.}$$

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, w = 22 mm **Ans.**

Thickness of key, t = 14 mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\,(d_b)^2\ 70 \times 6 \times 75 = 5830(d_b)^2$$

$\therefore$ $(d_b)^2 = 2865 \times 10^3 / 5830 = 492$ or $d_b = 22.2$ mm

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

Chapter 05: Design of a Closed Coil Helical Spring

Spring:

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

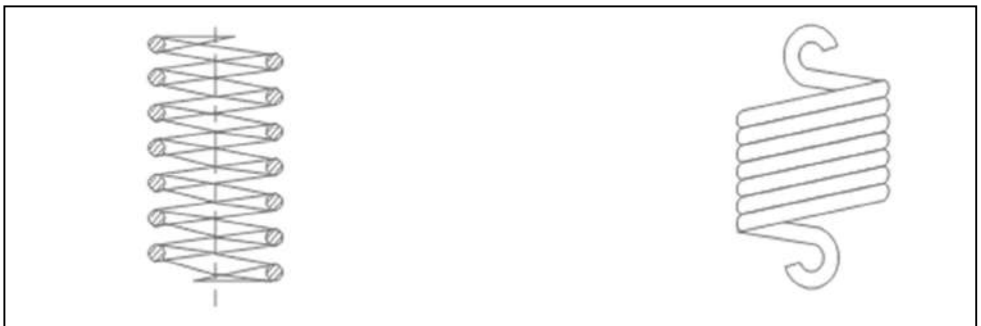
Applications of Springs:

- To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
- To apply forces, as in brakes, clutches and spring-loaded valves.
- To control motion by maintaining contact between two elements as in cams and followers.
- To measure forces, as in spring balances and engine indicators.
- To store energy, as in watches, toys, etc.

Types of Springs:

a) Helical Springs:

- The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile loads.
- The cross-section of the wire from which the spring is made may be circular, square or rectangular.
- The two forms of helical springs are compression and tension helical.
- The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion.



(*a*) Compression helical spring. (*b*) Tension helical spring.

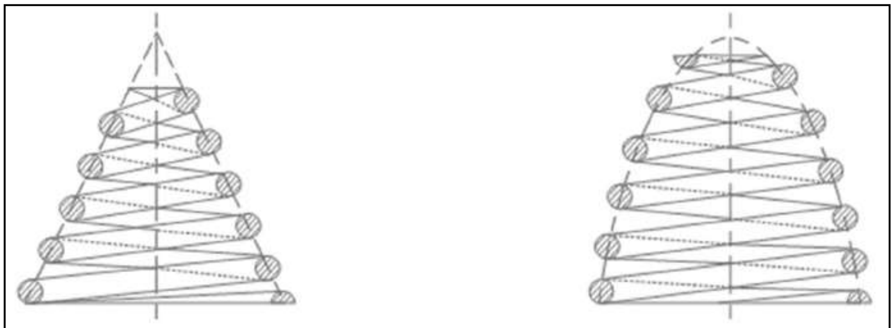
- In other words, in a closely coiled helical spring, the helix angle is very small; it is usually less than 10°.
- The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.
- In open coiled helical springs, the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large.

The helical springs have the following advantages:

- These are easy to manufacture.
- These are available in wide range.
- These are reliable.
- These have constant spring rate.
- Their performance can be predicted more accurately.
- Their characteristics can be varied by changing dimensions.

b) Conical and Volute springs:

- The conical and volute springs are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired.
- In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate.

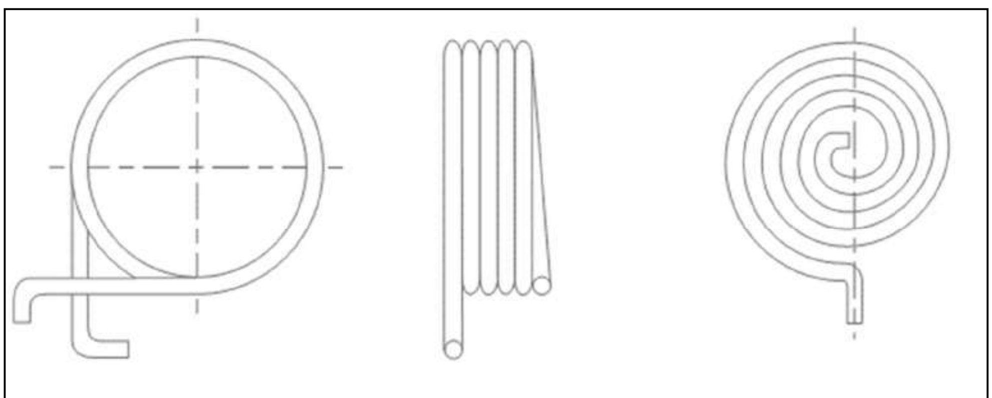


(*a*) Conical spring. (*b*) Volute spring.

- The major stresses produced in conical and volute springs are also shear stresses due to twisting.

c) Torsion Springs:

- These are used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms.
- The spiral type is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

- The major stresses produced in torsion springs are tensile and compressive due to bending.

d) Laminated or leaf springs:

- It consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts.
- These are mostly used in automobiles.
- The major stresses produced in leaf springs are tensile and compressive stresses.

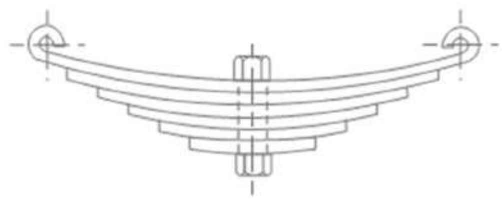


Fig. 23.4. Laminated or leaf springs.

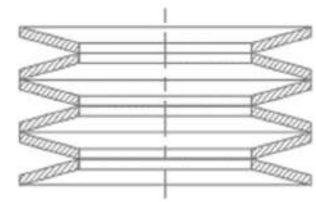


Fig. 23.5. Disc or bellevile springs.

e) Disc or Bellevile springs:

- These springs consist of a number of conical discs held together against slipping by a central bolt or tube.
- These springs are used in applications where high spring rates and compact spring units are required.

f) Special purpose springs:

- These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring.
- These springs are used for special types of application only.

Materials used for helical spring:

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the following services for which they are used such as

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

Table 23.1. Values of allowable shear stress, Modulus of elasticity and Modulus of rigidity for various spring materials.

Material	Allowable shear stress (τ) MPa			Modulus of rigidity (G) kN/m^2	Modulus of elasticity (E) kN/mm^2
	Severe service	Average service	Light service		
1. Carbon steel					
(a) Upto to 2.125 mm dia.	420	525	651		
(b) 2.125 to 4.625 mm	385	483	595		
(c) 4.625 to 8.00 mm	336	420	525		
(d) 8.00 to 13.25 mm	294	364	455		
(e) 13.25 to 24.25 mm	252	315	392	80	210
(f) 24.25 to 38.00 mm	224	280	350		
2. Music wire	392	490	612		
3. Oil tempered wire	336	420	525		
4. Hard-drawn spring wire	280	350	437.5		
5. Stainless-steel wire	280	350	437.5	70	196
6. Monel metal	196	245	306	44	105
7. Phosphor bronze	196	245	306	44	105
8. Brass	140	175	219	35	100

Standard size spring wire (SWG):

Table 23.2. Standard wire gauge (SWG) number and corresponding diameter of spring wire.

SWG	Diameter (mm)	SWG	Diameter (mm)	SWG	Diameter (mm)	SWG	Diameter (mm)
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

Terms used in Compression Springs:

a) Solid length:

When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

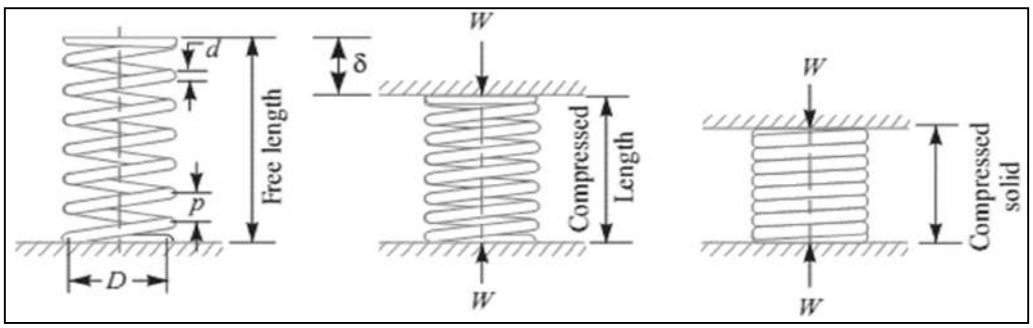
$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.

b) Free length:

The free length of a compression spring is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).



Mathematically,

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\ \delta_{max}$$

The following relation may also be used to find the free length of the spring, i.e.

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

c) Spring index:

The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire.

Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

d) Spring rate:

The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

e) Pitch:

The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $p = \text{Free length} / (n' - 1)$

The pitch of the coil may also be obtained by using the following relation, i.e.

Pitch of the coil, $p = [(L_F - L_S)/n'] + d$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted:

(a) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(b) The spring should not close up before the maximum service load is reached.

Note: In designing a tension spring, the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$$L_F = n.d + (n - 1)$$

and pitch of the coil, $p = L_F/(n-1)$.

End connection for tension helical spring:

The tensile springs are provided with hooks or loops which may be made by turning whole coil or half of the coil. In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.

The main disadvantage of tension spring is the failure of the spring when the wire breaks.

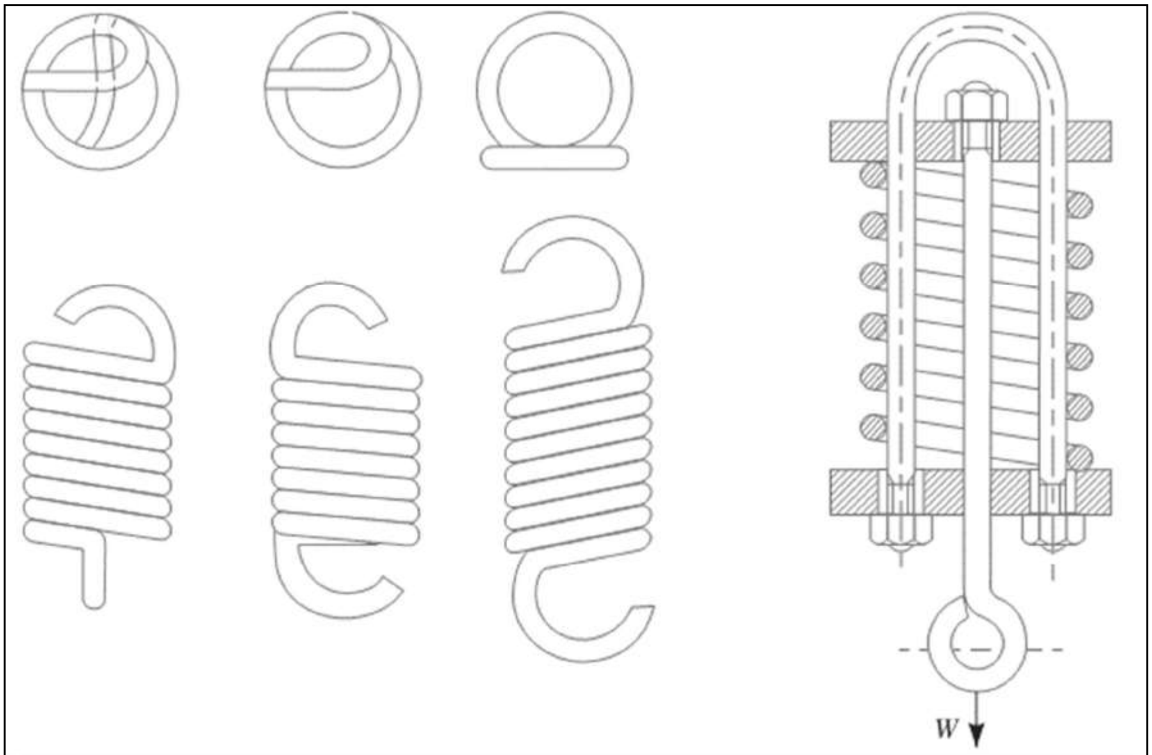


Fig. 23.8. End connection for tension helical springs.

Fig. 23.9. Compression spring for carrying tensile load.

Note: The total number of turns of a tension helical spring must be equal to the number of turns (n) between the points where the loops start plus the equivalent turns for the loops. It has been found experimentally that half turn should be added for each loop. Thus for a spring having loops on both ends, the total number of active turns,

$$n' = n + 1$$

Stresses in Helical Springs of Circular Wire:

Consider a helical compression spring made of circular wire and subjected to an axial load W.

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

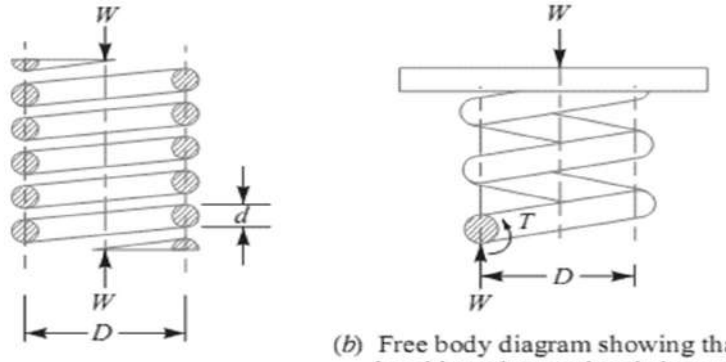
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.

(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring. The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \qquad \tau_1 = \frac{8W.D}{\pi\, d^3} \qquad \text{...(i)}$$

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire:

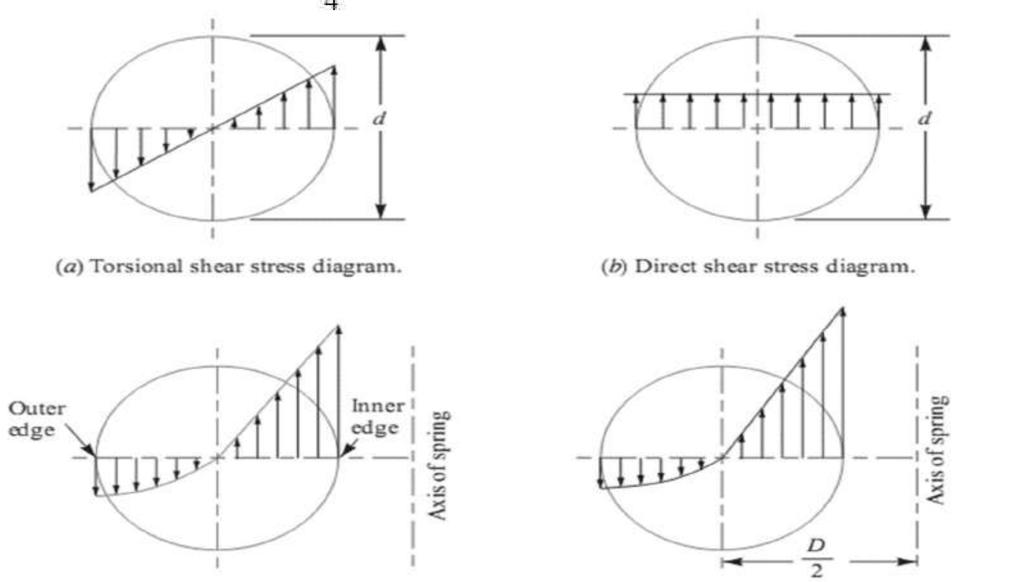
1. Direct shear stress due to the load W, and

2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi\, d^2} \qquad \text{...(ii)}$$



(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

The positive sign is used for the inner edge of the wire and negative sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\, d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\, d^3} \qquad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8\,WD}{\pi\, d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation **(iii)**. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A. M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (d).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\, d^3} = K \times \frac{8\,W.C}{\pi\, d^2} \qquad ...(iv)$$

where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig. 23.12.

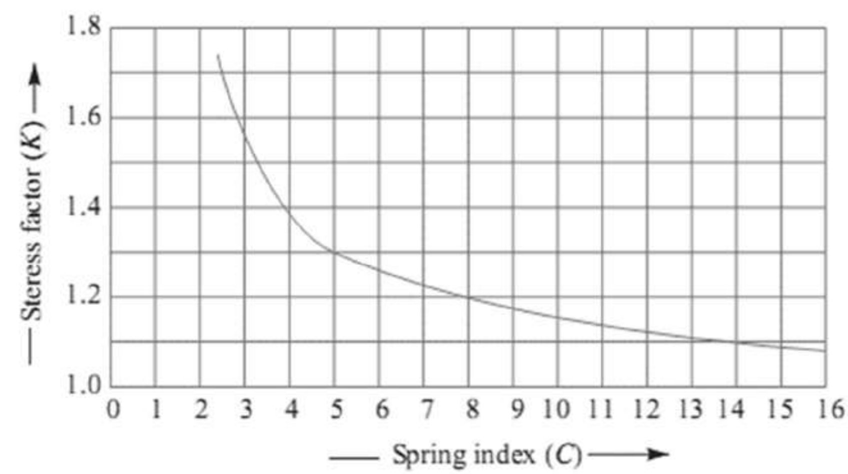


Fig. 23.12. Wahl's stress factor for helical springs.

Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that

$$K = K_S \times K_C$$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

Deflection of Helical Springs of Circular Wire:

Total active length of the wire,

l = Length of one coil × No. of active coils = π D × n

Let θ = Angular deflection of the wire when acted upon by the torque T.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \quad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \quad ...\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

= $(\pi/32) \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

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Substituting this value of θ in equation *(i)*, we have

$$\delta = \frac{16W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \quad ...(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

Eccentric Loading of Springs:

- Sometimes, the load on the springs does not coincide with the axis of the spring, i.e. the spring is subjected to an eccentric load.
- The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side.
- When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor D/(2e+D), where D is the mean diameter of the spring.

Surge in Springs:

- When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire.

- A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils.
- In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.
- If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur.
- This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called **surge**.

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\, G . g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,
D = Mean diameter of the spring,
n = Number of active turns,
G = Modulus of rigidity,
g = Acceleration due to gravity, and
ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods:

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : d = 6 mm ; D_o = 75 mm ; τ = 350 MPa = 350 N/mm² ; G = 84 kN/mm² = 84×10^3 N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index, $$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\, d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$$\therefore \qquad W = 350 / 0.848 = 412.7 \text{ N } \textbf{Ans.}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$$\therefore \qquad W = 350 / 0.913 = 383.4 \text{ N } \textbf{Ans.}$$

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

Example 23.5. *Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5.*

The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm².

Take Wahl's factor, $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$, *where C = Spring index.*

Solution. Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

$$\therefore \qquad d^2 = 16\,677 / 420 = 39.7 \quad \text{or} \quad d = 6.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm } \textbf{Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\ n$$

$$\therefore \quad n = 25 / 1.86 = 13.44 \text{ say } 14 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\ \delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

Example 23.6. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, $G = 84$ kN/mm^2.*

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm^2 ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$$\therefore \quad d^2 = 6875 / 82.48 = 83.35 \quad \text{or} \quad d = 9.13 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for $W = 500$ N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$$\therefore \qquad n = 6 / 0.63 = 9.5 \text{ say } 10 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

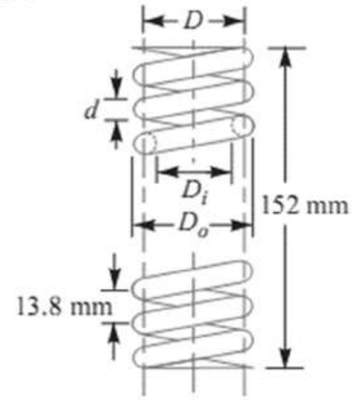

Fig. 23.14

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

The spring is shown in Fig. 23.14.

Que: *A load of 2 kN is dropped axially on a close coiled helical spring, from a height of 250 mm. The spring has 20 effective turns, and it is made of 25 mm diameter wire. The spring index is 8. Find the maximum shear stress induced in the spring and the amount of compression produced. The modulus of rigidity for the material of the spring wire is 84 kN/mm^2.*

Que: *Design a helical spring of round wire supports a static load of 1000 N. The inside diameter of the spring coil must not be smaller than 50 mm. The spring is to deflect 15 mm. The permissible shear stress in the spring material is 400 N/mm^2.*