LECTURE NOTES
ON
FLUID MECHANICS

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Chapter-1

Properties of Fluid

Fluid

Definition:

A fluid is a substance which is capable of flowing or a substance which deforms continuously when subjected to external shearing force.

Characteristics:

- It has no definite shape of its own but will take the shape of the container in which it is stored.
- A small amount of shear force will cause a deformation.

Classification:

A fluid can be classified as follows:

- Liquid
- Gas

Liquid:

It is a fluid which possesses a definite volume and assumed as incompressible

GAS:

It possesses no definite volume and is compressible.

Fluids are broadly classified into two types.

- Ideal fluids
- Real fluids

Ideal fluid:

An ideal fluid is one which has no viscosity and surface tension and is incompressible actually no ideal fluid exists.

Real fluids:

A real fluid is one which has viscosity, surface tension and compressibility in addition to the density.

PROPERTIES OF FLUIDS:

1. **density or mass density : (S)**

   Density of a fluid is defined as the ratio of the mass of a fluid to its vacuum. It is denoted by δ The density of liquids are considered as constant while that of gases changes with pressure & temperature variations.

   Mathematically

$$\rho = \frac{mass}{volume}$$

$$\text{Unit} = \frac{kg}{m^3}$$

$$\rho_{water} = 1000 \ \frac{kg}{m^3}$$

$$\text{or} \ \frac{gm}{cm^3}$$

2. **<u>Specific weight or weight density((W):</u>**

Specific weight of a fluid is defined as the ratio between the weights of a fluid to its valume. It is denoted by W.

Mathematically $W = \frac{weight\ of\ fluid}{volume\ of\ fluid}$

$= mg/v$

$W = \rho g$

Unit - $\frac{N}{m^3}$

3. **<u>Specific volume:</u>**

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

Mathematically

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

Unit: $\frac{m^3}{kg}$

4. **<u>Specific gravity:</u>**

Specific gravity is defined as the ratio of the weight density of a fluid to the density or when density standard fluid.

For liquids the standard fluid is water.

For gases the standard fluid is air.

It is denoted by the symbol S

Mathematically, $$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus weight density of a liquid $= S \times$ Weight density of water

$= S \times 1000 \times 9.81 \text{ N/m}^3$

The density of a liquid $= S \times$ Density of water

$= S \times 1000 \text{ kg/m}^3$.

Simple Problems:

Problem: - 1

Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (w)
$$= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \text{ N/m}^3.\ Ans.}$$

(*ii*) Density (ρ)
$$= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \text{ kg/m}^3.\ Ans.}$$

(*iii*) Specific gravity
$$= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= \mathbf{0.7135.\ Ans.}$$

Problem: - 2

Calculate the density, specific weight and specific gravity of one litre of petrol of specific gravity = 0.7

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Using equation (1.1.A),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = \mathbf{700 \text{ kg/m}^3.\ Ans.}$

(*ii*) *Specific weight* (w)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = \mathbf{6867 \text{ N/m}^3.\ Ans.}$

(*iii*) *Weight (W)*

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

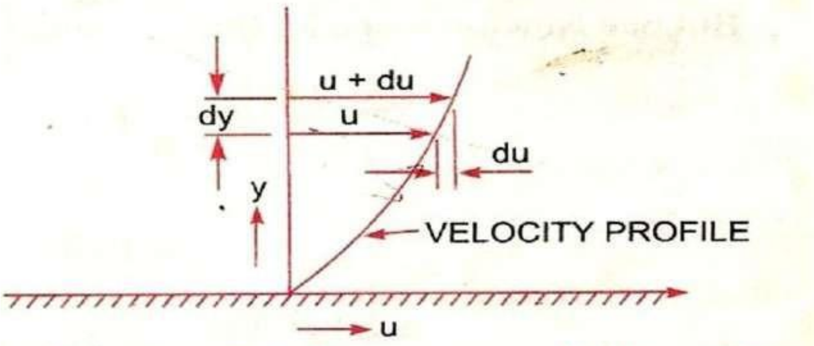
$$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

$$\therefore \quad W = 6867 \times 0.001 = \mathbf{6.867 \text{ N.}\ Ans.}$$

Viscosity:

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Let two layers of a fluid at a distance dy apart, move one over the other at different velocities u and u + du.



Velocity variation near a solid boundary.

The viscosity together with the with the relative velocity between the two layers while causes a shear stress acting between the fluid layers, the top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by r.

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Mathematically $r \; \alpha \frac{du}{dy}$

$$r = \mu \frac{du}{dy}$$

Where μ = co-efficient of dynamic viscosity or constant of proportionality or viscosity

$\frac{du}{dy}$ = rate of shear strain or velocity gradient

$$\mu = \frac{c}{\frac{du}{dy}}$$

If $\frac{du}{dy} = 1$,

then $\mu = r$

Viscosity is defined as the shear stress required to produce unit rate of shear strain.

Unit of viscosity in S.I system - $\frac{Ns}{m^2}$

in C.G.S – $\frac{Dyne\ s}{cm^2}$

in M.K.S. - $\frac{kgfs}{m^2}$

$\frac{Dyne\ s}{cm^2}$ = 1 Poise

1 $\frac{Ns}{m^2}$ = 10 poise

1 Centipoise = $\frac{1}{100}$ poise

Kinematic Viscocity:

It is defined as the ratio between the dynamic viscosity and density of fluid.

It is denoted by ϑ.

Mathematically

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad \text{...(1.4)}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{ \because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4}\ \text{m}^2/\text{s}$

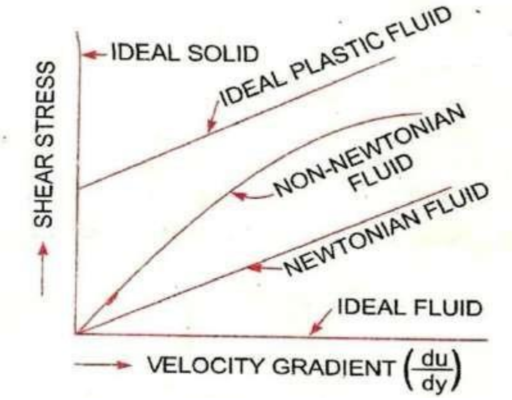
Centistoke means $= \frac{1}{100}$ stoke.

Newton's law of viscosity:

It states that the shear stress on a fluid element layer is directly proportional to the rate of shear stear strain. The constant of proportionality is called the co-efficient of viscosity.

Mathematically $\quad r = \mu \frac{du}{dy}$

Fluids which obey the above equation or law are known as Newtonian fluids & the fluids which do not obey the law are called Non-Newtonian fluids.



Surface tension:

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a stretched membrane under tension. The magnitude of this force per unit length of the free will has the same value as the surface energy per unit area.

It is denoted by σ

Mathematically $\sigma = \frac{F}{L}$

Unit in si system is N/m

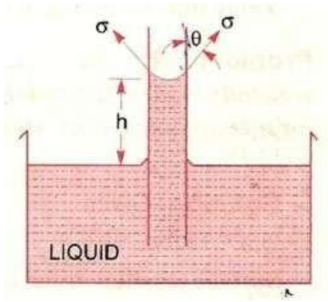
CGS system is Dyne/cm

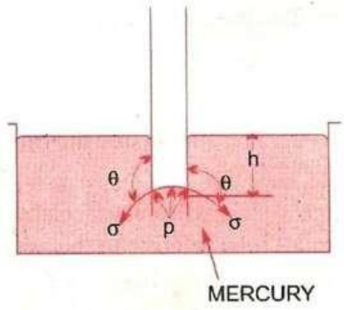
MKS system is kgf/m

Capillarity:

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is is known as capillary rise while the fall of the liquid surface is known as capillary depression.

It is expressed in terms of cm or mm of liquid





Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Chapter-2

Fluid Pressure And It's Measurements

Syllabus:

2.1 Definitions and units of fluid pressure, pressure intensity and pressure head
2.2 Concept of atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure
2.3 Pressure measuring instruments Manometers: Simple and differential Bourdon tube pressure gauge (Simple Numerical)

Pressure of a Fluid:

When a fluid is contained in a vessel, it exerts force at all points on the sides & bottoms of the container. The force exerted per unit area is called pressure.

If P = Pressure at any point

F = Total force uniformly distributed over an area

A = unit area

P = F/A

Unit of pressure - $\frac{kgf}{m^2}$ in M.K.S.

- $\frac{N}{m^2}$ in S.I.

- $\frac{Dyne}{cm^2}$

1pascal = 1N/m^2

1 kpa = 1000 N/m^2

Pressure head of a liquid:

A liquid is subjected to pressure due to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Let a bottomless cylinder stand in the liquid

Let w = specific weight of the liquid.

H = height of the liquid in the cylinder.

A = Area of the cylinder.

$$P = \frac{F}{A} = \frac{weight\ of\ the\ liquid\ in\ the\ cylinder}{Area\ of\ the\ cylinder}$$

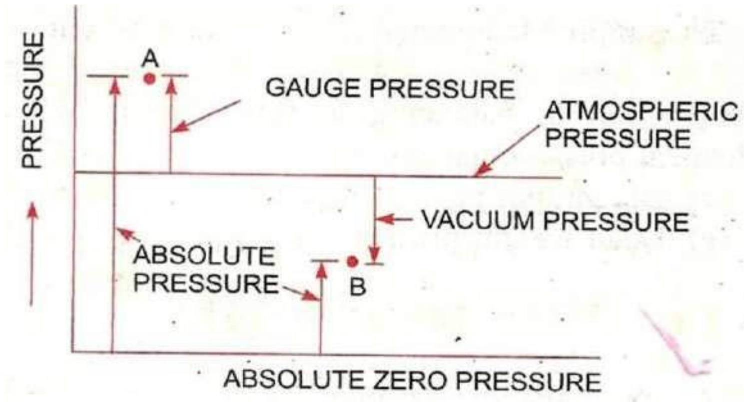
$$= \frac{W \times A\ h}{A}$$

= Wh

= ρgh

So intensity of pressure at any point in a liquid is proportional to its depth.

ABSOLUTE, GAGUE, ATOMOSPHERIC, AND VACCUME PRESSURES:



Atmospheric Pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which It is in contact & known as atmospheric pressure.

Absolute pressure:

It is defined as the pressure which is measured with reference to absolute vacuum pressure or absolute zero pressure.

Gauge pressure:

It is defined as the pressure which is measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Vacuum pressure:

It is defined as the pressure below the atmospheric pressure.

Mathematically:

Absolute pressure = Atmospheric pressure + gauge pressure

Or $P_{abs} = P_{atm} + P_{gauge}$

Vacuum pressure = Atmospheric pressure – Absolute pressure

$P_{vacuum} = P_{atm} - P_{abs}$

Pressure Measuring Instruments:

The pressure of a fluid is measured by the following devices :

1. **Manometers**
2. **Mechanical Gauges**.

Manometers:

Manometers are defined as the device used for measuring the pressure at a point in a fluid by balancing the collomn of fluid by the same another column of the fluid. They are classified as:

(a) **Simple manometers.**
(b) **Differential Manometers**.

Mechanical Gauges:

Mechanical gauges are defined as the device used for measuring the pressure by balancing the fluid column by the spring or dead weight. Commonly used mechanical pressure gauges are :

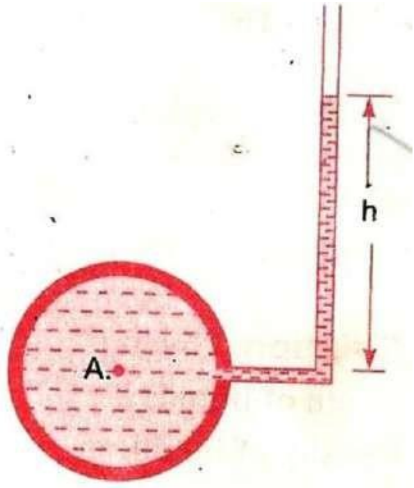
- **Diaphragm pressure gauge**
- **Bourdon tube pressure gauge**
- **Dead –weight pressure gauge**
- **Bellow pressure gauge**

Simple Manometres:

A simple manometer of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

- **Piezometer**
- **U- tube Manometer**
- **Single Column Manometer**

Piezometer:

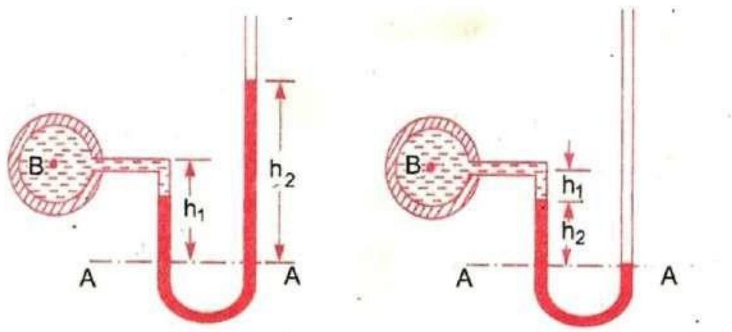


It is the simple form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Figure. The rise of liquid gives the pressure head at that point A. Then pressure at A

$$P_A = \rho gh$$

U – tube Manometer:

It consist of glass tube bent in U- shape , one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure. The tube generally contains mercury.



(a) For gauge pressure (b) For vacuum pressure

(a) For Gauge Pressure:

Let be is the point which is to be measured, whose value is p. The datum line is A-A.

Let h_1= Height of light liquid above the datum line

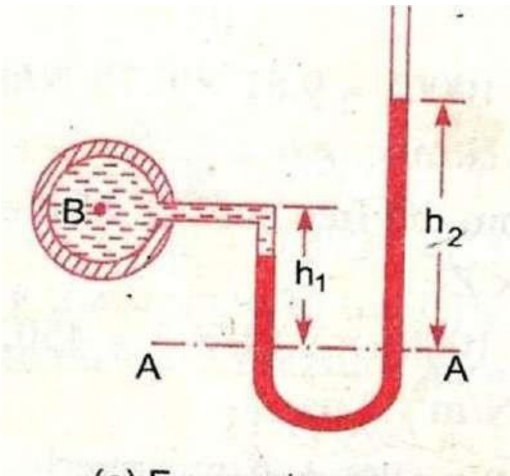
h_2= Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = Density of light liquid = 1000× S_1

S_2 = Sp. Gr. Of heavy weight

ρ_2 = density of heavy weight = 1000×S_2



(a) For gauge pressure

Pressure is same in a horizontal surface. Hence pressure above the horizontal datum surface line A-A in the left column and in the right column of U-tube manometer should be same pressure above A-A in the left column

$$= p_A + \rho_1 \times g \times h_1$$

Pressure above A-A in the right column

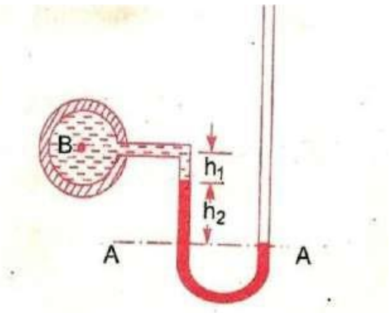
$$= \rho_2 \times g \times h_2$$

Hence equating the two pressures

$$p_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$p_A = (\rho_2 g h_2 - \rho_1 g h_1).$$

(b) For Vacuum Pressure:



For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in figure. Then Pressure above A-A in the left column

$$= \rho_2 g h_2 + \rho_1 g h_1 + p_A$$

Pressure head in the right column above A - A = 0

$$\rho_2 g h_2 + \rho_1 g h_1 + p_A = 0$$

$$p_A = -(\rho_2 g h_2 + \rho_1 g h_1)$$

Single Column Manometer:

Single column Manometer is modified form of a U- tube manometer in which a reservoir, having a large cross- sectional area (about 100 times as compared to the area of the tube) is connected to one of the limbs (say left limb)of the manometer as shown in figure. Due to large cross- sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

- **Vertical Single Column Manometer**
- **Inclined Single Column Manometer**

1. Vertical Single Column Manometer:

Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

H_2 = rise of heavy liquid in right limb

H_1 = height of center of pipe above X-X

P_A = Pressure at A, which is to be measured

A = Cross – sectional area of the reservoir

a = Cross sectional area of the right limb

S_1 = Sp.gr.of liquid in pipe

S_2 = Sp.gr. of heavy weight liquid in reservoir and right limb

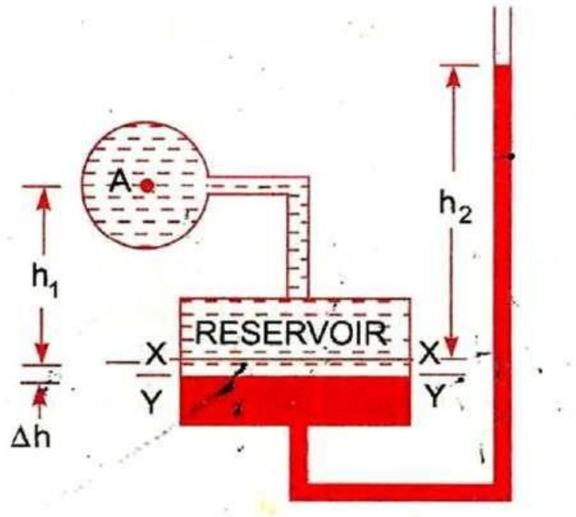
P_1 = Density in liquid in pipe

P_2 = Density of liquid in the reservoir

Fall of heavy liquid in the reservoir will cause a rise of heavy liquid level in the right limb.

$\therefore \quad A \times \Delta h = a \times h_2$

$\therefore \quad \Delta h = \frac{a \times h}{A}$ ....(i)



Now consider the datum line Y-Y as shown in Fig 2.15.Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in left limb above Y-Y $= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating the pressure, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = \rho_2 g\,(\Delta h + h_1) - \rho_1 g\ (\Delta h + h_1)$$

$$= \Delta h\,[\rho_2 g - \rho_1 g] + h_2\,\rho_2 g - h_1\,\rho_1 g$$

But from equation (i), $\Delta h = \dfrac{a \times h}{A}$

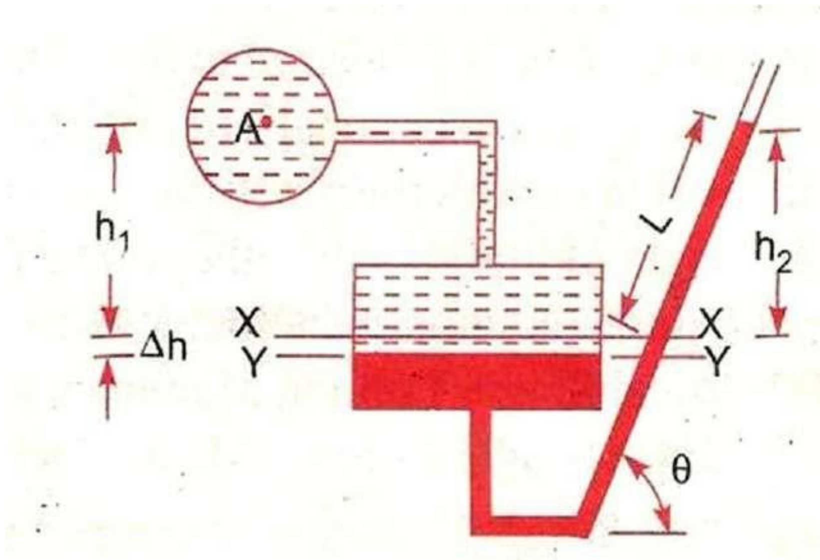
$$\text{So, } P_A = \frac{a \times h}{A}[\rho_2 g - \rho_1 g] + h_2\,\rho_2 g - h_1\,\rho_1 g$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

Then $P_A = h_2\,\rho_2 g - h_1\,\rho_1 g$

2. Inclined Single Column Manometer:

The given figure shows the inclined single column manometer which is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.



Let L = length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-X

$= L \times \sin\theta$

From the above equation for the pressure in the single column manometer the pressure at A is

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g .$$

Substituting the value of h_2, we get

$$P_A = \sin\theta \rho_2 g L - h_1 \rho_1 g .$$

DIFFERENTIAL MANOMETERS:

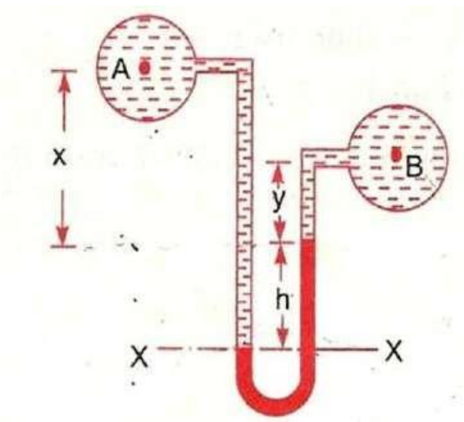
Differential manometers are the device use for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U- tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly used differential manometers are :

1. **U-tube differential manometer**
2. **Inverted U-tube differential manometer**

U-tube differential manometer:

Two points A and B are at different level

The given figure shows the differential manometers of U-tube type.



Let the two points A and B are at different level also contains liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B.

Let h = Difference of mercury level in the U- tube.

y = Distance of the center of B, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X .

Pressure above X-X in the limb

$= \rho_1 g(h + x) + P_A$

Where pressure P_A = Pressure at A.

Pressure above X-X in the right limb

$= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

Where pressure p_B = pressure at B.

Equating the two pressure, we have

$P_1 g(h + x) + P_A = p_g \times g \times h + p_2 g y + p_B$

∴ $P_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$

$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

∴ Different of pressure at A and B

$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

Two points A and B are at same level

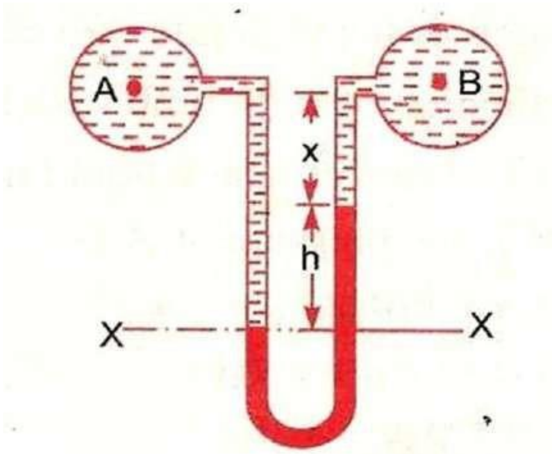
In the given figure A and B are the same level and contains the same liquid of density ρ_1, then

Pressure above X-X in right limb

$= \rho_g \times g \times h + \rho_1 \times g \times X + p_B$

Pressure above X-X in left limb

$= P_1 \times g \times (h + x) + P_A$



Equating the two pressure

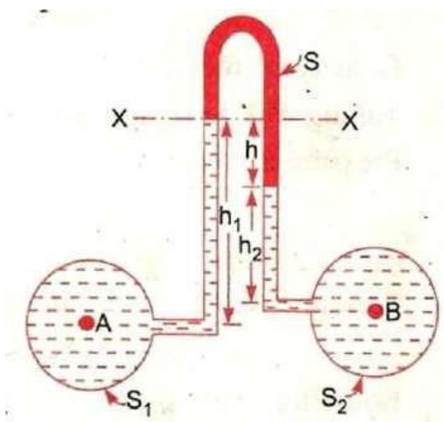
$p_g \times g \times h + P_1 \times g \times X + p_B = P_1 \times g \times (h + x) + P_A$

$\therefore \quad P_A - p_B = P_g \times g \times h + P_1 g x - P_1 g \times (h + x)$

$= g \times h\,(P_g - P_1)$

Inverted U-tube Differential Manometer:

It consists of an inverted U-tube, containing a light liquid. The two ends of the U-tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig 2.21 shows an inverted U-tube differential manometer connected to the points A and B. Let the pressure at A is more than the pressure at B.



Let h_1=Height of liquid in the left limb bellow the datum line X-X

h_2= Height of liquid in the right limb

h= Difference of light liquid

p_1=Density of liquid at A

p_2=Density of liquid at B

p_s= Density of light liquid

p_A=*Pressure at A*

p_B= Pressure at B.

Taking X-X datum line.

Then pressure in the left limb below X-X

$$= P_A - \rho_1 \times g \times h_1.$$

Pressures in the right limb below X-X

$$= P_B - \rho_2 \times g \times h_2 - \rho_S \times g \times h$$

Equating the two pressure

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_S \times g \times h$$

$$P_A - P_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_S \times g \times h$$

Bourdon's Tube Pressure Gauge:

- The pressure above or below the atmospheric pressure may be easily measured with the help of Burdon tube pressure gauge.
- It consists of an elliptical tube ABC bent into an arc of a circle. This bent up tube is called Burdon tube.
- When the gauge tube is connected to the C, the fluid under pressure flows into the tube the bourdon tube as a result of the increased pressure tends to straighten itself.
- Since the tube is encased in a circular cover therefore.it tends to become circular instrad of straight.
- The elastic beforemation of the bourdon rotates the pointer.
- The pointer moves over a calibrates which directly gives the pressure.

Numerical problems:

The right limb of asimple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which afluid of sp gravity 0.9 isflowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the deference of mercury level inthe two limbs is 20 cm.

A single column manometer is connected to a pipe containing a liquied of sp. Gravity 0.9 find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for manometer reading. The sp. Gravity of mercury is 13.6 .

a deferential manometer is connected at the two points A and B of two pipes. The pipe A contains aliquis of sp. Gravity =1.5 wile pipe B containsa liquid of sp. Gravity 0.9 the pressure at A and B are 1kg/ cm^2 and 1.80 kg/cm^2 respectively. Find the deference in mercury level in the deferential manometer.

Q.4water is flowing through two deference pipes to which an inverted deferential manometer having an oil of sp. Gravity 0.8 is connected. The pressure head in the pipe A is 2m of water, find the pressure in the pipe B for the manometer readings.

Chapter-3

Hydrostatics

Syllabus:

Definition of hydrostatic pressure
Total pressure and centre of pressure on immersed bodies (Simple Numericals)
Archimedis' principle, concept of buoyancy, metacentre and metacentric height
Concept of floatation

Hydrostatics:

Hydrostatics means the study of pressure exerted by thye liquid at rest & the direction of such a pressure is always right angle to the surface on which it acts.

Total pressure and center of pressure:

Total pressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with surfaces. This force always acts normal to the surface.

Center of pressure:

Center of pressure is defined as the point of application of the total pressure on the surface.

There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined. The submerged surfaces may be:

1. **Vertical plane surface**
2. **Horizontal plane surface**
3. **Inclined plane surface**
4. **Curved surface.**

Vertical plane surface submerged in liquid

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in figure

Let A = total area of the surface

H = distanced of C.G. of the area from free surface of liquid

G = center of gravity ofplane surface

P = center of pressure

h* = distance of center of pressure from free surface of liquid.

Total pressure(F):

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on surface is then calculated by integrating the force on small strip.

Consider a strip of thickness dh & width b at a depth of h form free surface of liquid.

Pressure intensity on the strip

$$p = \rho gh$$

Area of the strip, $dA = b \times dh$

Total pressure forceon strip, $dF = \rho dA$

$= \rho gh \times b \times dh$

Total pressure force on thge whole surface

$F = \int dF = \int \rho gh \times b \times dh$

$= \rho g \int h \times b \times dh$

$\int h \times dA$ = moment of surface area about the free surface of liquid

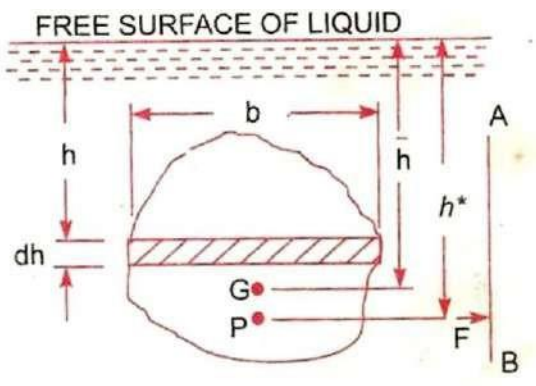
= Area of surface × distance of C.G. from the free surface

$= A \times \bar{h}$

So, $F = \rho g A \bar{h}$

<u>Centre of the pressure:(</u> h*)

Centre of pressure is calculated by using the principle of moments which states that the moment of resultant force about an axis is equal to the sum of moments of the components about the same axis.



The resultant force F is acting at P, at a distance **h*** from the free surface of liquid.

Hence moment of force F about free surface of liquid = F × h*

But moment force dF acting on a strip about the free surface of liquid = dF × h

Sum of moments of all such forces about free surface of liquid

$$= \int qgh \times b \times dh \times h$$

$$= qg \int h \times b \times dh \times h$$

$$= qg \int bh^2 \, dh$$

$$= qg \int h^2 \, dA$$

$\int h^2 \, dA$ = moment of inertia of the surface area about the free surface of liquid = Io

Sum of the moments about free surface

$$= qg \, Io$$

$$F \times h^* = qg \, Io$$

$$qgA\bar{h} \times h^* = qg \, Io$$

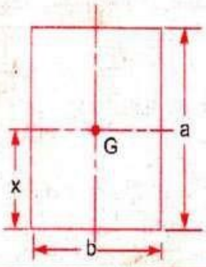
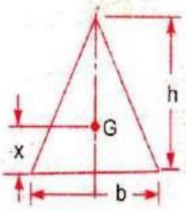
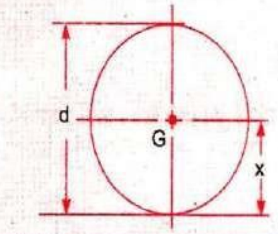
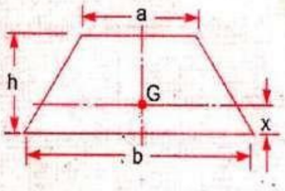
$$h^* = \frac{qgIo}{qgAh}$$

$$= \frac{Io}{Ah}$$

By the parallel axis theorem, we have

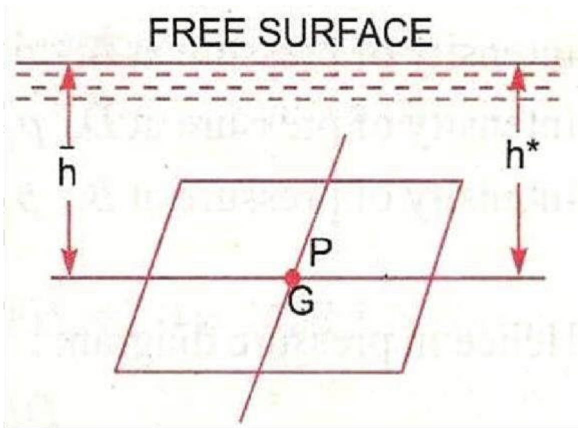
$$Io = I_G + A \times (\bar{h})^2$$

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Horizontal plane surface submerged in liquid:

Consider a plane horizontal surface immersed in a static fluid as every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface.



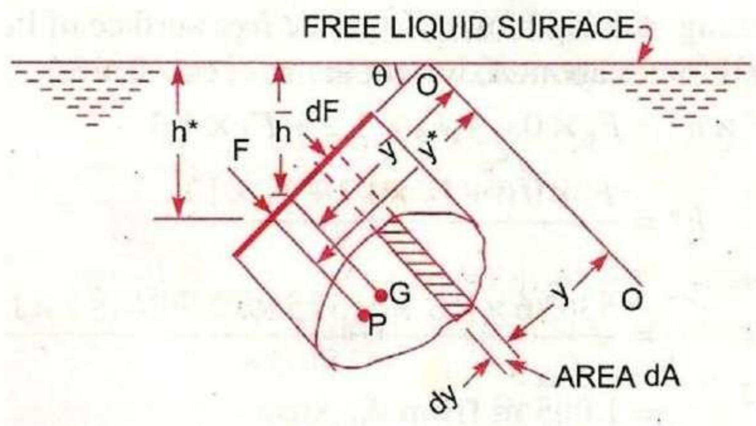
$P = \rho g\bar{h}$

A = total area

F = P × A

$= \rho g A\bar{h}$

Inclined plane surface submerged in liquid:



Let A = total area of the inclned surface

H = depth of C.G. of inclined area from free surface.

h* = distance of center of pressure from free surface of liquid.

8 = angle made by the plane of surface with free liquid surface.

Let the plane of the surface if produced meet the free liquid surface at 0. Then 0-0 is the axis parallel to the plane of the surface

$\bar{y}$ = distance of C.G of the inclined surface from 0-0.

y* = distance of the centre of pressure from 0-0.

Consider a small strip of area dA at a depth 'h' from free surface & at a distance y from axis 0-0.

$$P = qgh$$

$$dF = pdA$$

$$= qgh\ dA$$

Total pressure force

$$F = \int dF = \int qgh\ dA$$

$$h = ysin8$$

$$F = \int qgysin8\ dA$$

$$= qgsin8 \int ydA$$

$$= qgsin8\ Io$$

$$= qgsin8 A\ \bar{y}$$

$$= qgA\ \bar{y} sin8$$

$$= qgA\bar{h}$$

Centre of pressure:

Pressure force on the strip $dF = qgh\,dA$

$$= qgy\sin 8\,dA$$

Moment of the force dF about 0 – 0

$$= dF \times y = qgy^2\sin 8\,dA$$

Sum of moments of all such forces about 0 – 0

$$= qg\sin 8\, y^2 dA$$

$\int y^2\,dA$ = moment of inertia of the surface about 0 - 0 = Io

$$= qg\sin 8 I_o$$

Moment of total force about 0 – 0

$$= F\,y^*$$

$$F\,y^* = qg\sin 8\, I_o$$

$$qgA\,\bar{h} \times \frac{h^*}{\sin 8} = qg\sin 8\, I_o$$

$$h^* = \frac{cin^2 8}{Ah} I_o$$

$$= \frac{cin^2 8}{A\,\bar{h}}\left[I_G + A \times (\bar{y})^2\right]$$

Here $\frac{-}{\bar{y}} = \sin 8$

$$\bar{y} = \frac{\bar{h}}{\sin 8}$$

$$h^* = \frac{cin^2 8}{A\,\bar{h}}\left[I_G + A \times \left(\frac{\bar{h}}{\sin 8}\right)^2\right]$$

$$h^* = \frac{I_G\,cin^2 8}{A\bar{h}} + \bar{h}$$

Archimedes principle:

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of fluid displaced by the body.

Buoyancy:

Whenever a body is immersed wholly or partially in a fluid it is subjected to an upword force which tends to lift itup. This tendency for an immersed body to be lifted up in the fluid due to an upward force opposite to action of gravity is known as buoyancy this upward force is known as force of buoyancy.

Centre of Buoyancy:

It is defined as the point through which the forced of buoyancy is supposed to act. The force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body.

Canter of buoyancy will be the centre of gravity of the fluid displaced.

Problem-1:

Find the volume of the water displaced & position of centre of duoyancy for a wooden block of width 2.5m & of depth 1.5m when it flats horizontally in water. The density of wooden block is 6540 kg/m3.& its length 6.0m.

Solution:

Width = 2.5 m

Density of wooden block = 650kg/m^3

Depth = 1.5m

Length = 6m

Volume of the block

$$= 2.5 \times 1.5 \times 6$$

$$= 22.50 m^3$$

Volume of the block = Wt of water displaced

$$= W \times V$$

$$= qg \times V$$

$$= 650 \times 9.81 \times 6$$

$$= 143471 \text{ N}$$

Volume of water displaced

$$= \frac{weight}{qw \times g}$$

$$= \frac{143471}{1000 \times 9.81}$$

$$= 14.625 m^3$$

Position of centre of buoyancy

Volume of wooden block in water = volume of water displaced

$$2.5 \times 6 \times h = 14.625$$

$$\Rightarrow h = \frac{14.625}{2.5 \times 6}$$

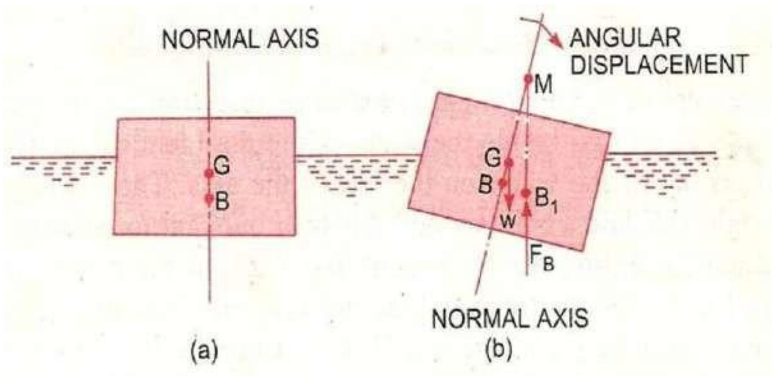
$$= 0.975 m$$

Centre of buoyancy $= \frac{0.975}{2}$

$= 0.4875$ m from base.

Meta-centre;

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The mate centre may also be defined as the point at which the lme of action of the force of buoyancy will melt the normal axis. Of the body when the body is given a small angular displacement.



Mate centre height:

The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height i.e the distance MG.

Concept of flotation:

Flotation:

When a body is immersed in any fluid, it experiences two forces. First one is the weight of body W acting vertically downwards, second is the buoyancy force F_b acting vertically upwards in case W is greater than F_b, the weight will cause the body to sink in the fluid. In case W = F_b the body will remain in equilibrium at any level. In case W is small than F_b the body will move upwards in fluid. The body moving up will come to rest or top moving up in fluid when the fluid displaced by it's submerged part is equal to its weight W, the body in this situation is said to be floating and this phenomenon is known as flotation.

Principle of flotation:

The principle of flotation states that the weight of the floating body isequal to the weight of the fluid displaced by the body.

Consider a body floating at the free surface of the liwuid. The shaded part of the body is inside the fluid and it has volume V_1 The other part of the body is in air and it has volume V_2. Now the body can be considered to be in two fluids viz. air and liquid. Hence buoyant force

$$F_b = \rho_{liquid} V_1 g_1 + \rho_{air} V_2 g_2 = W$$

Since $\rho_{air} \ll \rho_{liquid}$

$$F_b = \rho_{liquid} V_1 g = W$$

Buoyancy force is equal to weight of the liquid displaced

The ways to make the body float:

The body can be made to float:

1. Decreasing the weight of the body while keeping the volume same. For example, making body hollow.
2. Increasing the volume of the body while keeping the body same. For example, attaching live jacket to a person fixed the person floating.

Chapter-4

KINEMATICS

Syllabus:

Types of fluid flow **Continuity equation (Statement and proof for one dimensional flow)** **Bernoulli's theorem (Statement and proof)** **Applications and limitations of Bernoulli's theorem (Venturimeter, pitot tube)** **(Simple Numerical)**

Introduction:-

This chapter includes the study of forces causing fluid flow. The dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

TYPES OF FLOW:-

The fluid flow is classified as follows:

- **STEADY AND UNSTEADY FLOW**
- **UNIFORM AND NON- UNIFORM FLOWS**
- **LAMINAR AND TURBULANT FLOWS**
- **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS**
- **ROTATIONAL AND IRROTATIONAL FLOWS**
- **ONE, TWO, THREE DIMENSIONAL FLOW**

➢ **STEADY AND UNSTEADY FLOW**:-

1. **Steady flow**:-

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density at a point do not change with time.

Thus, mathematically

$$\left(\frac{6v}{6t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{6p}{6t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{6\rho}{6t}\right)_{0,y_0,z_0} = 0$$

Where x_0, y_0, z_0 is a point in fluid flow .

2. **Unsteady flow:-**

Unsteady flow is defined as that type of flow in which the velocity, pressure, and density at a point changes w.r.t time.

Thus, mathematically

$$\left(\frac{6v}{6t}\right)_{0,y_0 z_0} \neq 0,$$

$$\left(\frac{6p}{6t}\right)_{0,y_0 z_0} \neq 0,$$

$$\left(\frac{6\rho}{6t}\right)_{0,y_0 z_0} \neq 0$$

- **UNIFORM AND NON- UNIFORM FLOWS:-**

1. **Uniform flow:-**

It is defined as the flow in which velocity of flow at any given time does not change w.r.t length of flow or space.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} = 0$$

where ∂v = velocity of flow ,

∂s = length of flow ,

T = time

2. **Non- uniform flows:-**

It is defined as the flow in which velocity of flow at any given time changes w.r.t length of flow.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} \neq 0$$

- **<u>LAMINAR AND TURBULANT FLOWS</u>**:-

1. **<u>Laminar flow</u>**:-

Laminar flow is that type of flow in which the fluid particles are moved in a well defined path called streamlines. The paths are parallel and straight to each other.

2. **<u>Turbulent flow</u>**:-

Turbulent flow is that type of flow in which the fluid particles are moved in a zig-zag manner.

For a pipe flow the type of flow is determined by Reynolds number (R_e)

Mathematically

$$R_e = \frac{VD}{v}$$

Where V = mean velocity of flow

D = diameter of pipe

V = kinematic viscosity

If R_e< 2000, then flow is laminar flow.

If R_e> 4000, then flow is turbulent flow.

If R_e lies in between 2000 and 4000, the flow may be laminar or turbulent.

- **<u>COMPRESSIBLE AND INCOMPRESSIBLE FLOWS</u>** :-

1. **<u>Compressible flow</u>**:-

Compressible flow is that type of flow in which the density of fluid changes from point to point.

So, $\partial \neq$ constant.

2. **<u>Incompressible flow</u>**:-

Incompressible flow is that type of flow in which the density is constant for the fluid flow.

So, $\partial =$constant

- **ROTATIONAL AND IRROTATIONAL FLOWS**:-

1. **Rotational flow:-**

Rotational flow is that of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

2. **Ir-rotational flow**:-

Irrotational flow is that type of flow in which the fluid particles while flowing along streamlines do not rotate about their own axis.

- **ONE, TWO, THREE DIMENSIONAL FLOW**:-

1. **One dimensional flow**:-

One dimension flow is defined as that type of flow in which velocity is a function of time and one space co-ordinate only.

For a steady one dimensional flow, the velocity is a function of one space co-ordinate only.

So, $U = f(x)$,

$V = 0$,

$W = 0$

U, V, W are velocity components in x, y, z direction respectively.

2. **Two-dimensional flow**:-

Two-dimensional flow is the flow in which velocity is a function of time and 2- space co- ordinates only. For a steady 2- dimensional flow the velocity is a function of two – space co-ordinate only.

So, $U = f_1(x,y)$,

$V = f_2(x,y)$,

$W = 0$

3. **Three-dimensional flow**:-

Three – dimensional flow is the flow in which velocity is a function of time and 3- space co-ordinates only. For steady three- dimensional flow, the velocity is a function of three space co-ordinates only.

So $U = f_1(x, y, z)$

$V = f_2(x, y, z)$

$W = f_3(x, y, z)$

RATE OF FLOW OR DISCHARGE

It is defined as the quantity of a fluid flowing per second through a section of pipe.

For an incompressible fluid the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

$$Q = A \cdot V$$

Where A = cross sectional area of the pipe

V = velocity of fluid across the section

Unit:-

1. For incompressible fluid

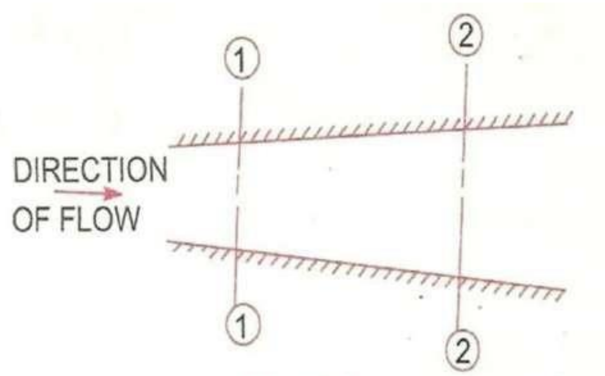
$$\frac{m3}{sec} \text{ or } \frac{litre}{sec}$$

2. For compressible fluid:

$$\frac{newton}{sec} \text{ (S.I units), } \frac{kgf}{sec} \text{ (M.K.S units)}$$

EQUATION OF CONTINUITY:-

It is based on the principle of conservation of mass. For a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.



Fluid flowing through a pipe.

Let V_1= average velocity at cross-section 1-1.

ρ_1 = density at cross-section 1-1

A_1 = area of pipe at section 1-1

V_2= average velocity at cross-section 2-2

ρ_2 = density at cross-section 2-2

A_2 = area of pipe at section 2-2

The rate of flow at section 1-1 =$\rho_1 A_1 V_1$

The rate of flow at section 2-2 =$\rho_2 A_2 V_2$

According to laws of conservation of mass rate of flow at section 1-1 is equal to the rate of flow at section 2-2 ,

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

This is called continuity equation.

If the fluid is compressible, then $\rho_1 = \rho_2$,

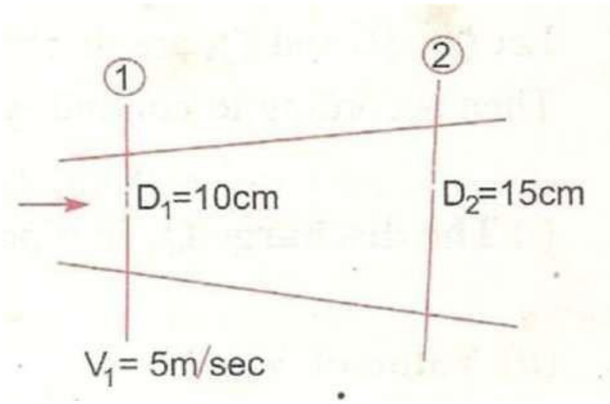
so $A_1 V_1 = A_2 V_2$

"If no fluid is added removed from the pipe in any length then the mass passing across different sections shall be same"

Simple Problems

Problem:-1

The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of the water flowing through the pipe at section 1 is 5m/s. Determine also the velocity at section 2.



Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1{}^2) = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

(*i*) Discharge through pipe is given by equation (5.1)

or
$$Q = A_1 \times V_1$$
$$= .007854 \times 5 = \mathbf{0.03927 \text{ m}^3/\text{s. Ans.}}$$

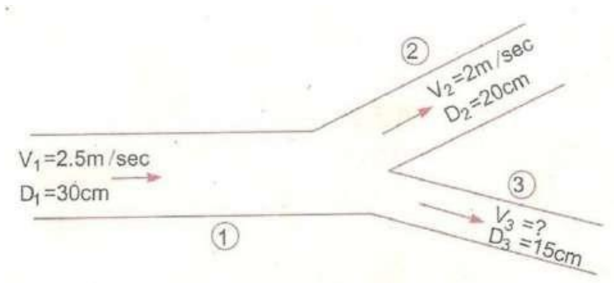
Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) ∴
$$V_2 = \frac{A_1V_1}{A_1} = \frac{.007854}{.01767} \times 5.0 = \mathbf{2.22 \text{ m/s.}}$$

Problem:-2

A 30m diameter pipe conveying water branches into two pipes of diameter 20cm and 15cm respectively. If the average velocity in the 340cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s

Solution:



Given Data:

D_1 = 30cm = 0.30m

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2$

$V_1 = 2.5$ m/s

D_2 = 20cm = 0.2m

$A_2 = \frac{\pi}{4} 0.2^2 = 0.0314\ m^2$

$V_2 = 2$m/s

D_3 = 15cm = 0.15m

$A_3 = \frac{\pi}{4} 0.15^2 = 0.01767\ m^2$

Let Q_1, Q_2, Q_3 are discharges in pipe 1, 2, 3 respsctively

$$Q_1 = Q_2 + Q_3$$

The discharge Q_1 in pipe 1 is given as

$Q_1 = A_1 V_1$

$= 0.07068 \times 2.5\ m^3/s$

$Q_2 = A_2 V_2$

$= 0.0314 \times 2.0\ 0.0628\ m^3/s$

Substituting the values of Q_1 and Q_2 on the above equation we get

$0.1767 = 0.0628 + Q_3$

$Q_3 = 0.1767 - 0.0628$

$= 0.1139$ m3/s

Again $Q_3 = A_3 V_3$

$= 0.01767 \times V_3$

Or $0.1139 = 0.01767 \times V_3$

$$V_3 = \frac{0.1139}{0.01767}$$

$= 6.44$m/s

Problem:-3

A 25 cm diameter pipe carries oil of sp. Gr. 0.9 at a velocity of 3m/s. At another section the diameter is 20cm. Find the velocity at this section and also mass rater of flow of oil.

Solution. Given :

at section 1, $D_1 = 25 \text{ cm} = 0.25 \text{ m}$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .25^2 = 0.049 \text{ m}^3$$

$$V_1 = 3 \text{ m/s}$$

at section 2, $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_2 = ?$$

Mass rate of flow of oil = ?

Applying continuity equation at sections 1 and 2,

$$A_1 V_1 = A_2 V_2$$

or $$0.049 \times 3.0 = 0.0314 \times V_2$$

$\therefore$ $$V_2 = \frac{0.049 \times 3.0}{.0314} = \textbf{4.68 m/s. Ans.}$$

Mass rate of flow of oil $= \text{Mass density} \times Q = \rho \times A_1 \times V_1$

Sp. gr. of oil $= \dfrac{\text{Densit of oil}}{\text{Densit of water}}$

$\therefore$ Density of oil $= \text{Sp. gr. of oil} \times \text{Density of water}$

$= 0.9 \times 1000 \text{ kg/m}^3 = \dfrac{900 \text{ kg}}{\text{m}^3}$

$\therefore$ Mass rate of flow $= 900 \times 0.049 \times 3.0 \text{ kg/s} =$ **132.23 kg/s. Ans.**

Bernoulli's equation:

Statement: It states that in a steady ideal flow of an in compressible fluid, the total energy at any point of flow is constant.

The total energy consists of pressure energy, kinetic energy & potential energy or datum energy. These energies per unit weight are

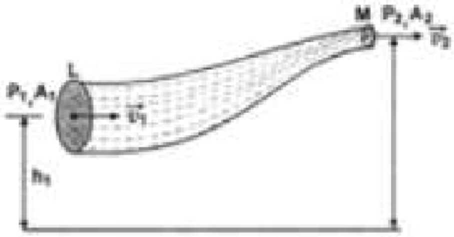
Pressure energy = $\frac{P}{\rho g}$

Kinetic energy = $\frac{v^2}{\rho g}$

Datum energy = z

Mathematically

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{Constant}$$



Proof: Let us consider the ideal liquid of density ρ flowing through the pipe LM of varying cross-section. Let P_1 and P_2 be the pressures at ends L and M and A_1 and A_2 be the areas of cross-sections at ends L and M respectively. Let the liquid enter with velocity V_1 and leave with velocity v_2. Let $A_1 > A_2$. By equation of continuity,

$$A_1 v_1 = A_2 v_2$$

Since $A_1 > A_2$,

$\therefore \quad v_2 > v_1 \quad$ and $\quad P_1 > P_2$

Let m be mass of liquid entering at end L in time t. In time t, the liquid will cover a distance of $v_1 t$.

Therefore the work done by pressure on the liquid at end L in time t is

$$W_1 = \text{force} \times \text{displacement}$$

$$= P_1 A_1 v_1 t \qquad \ldots(1)$$

Since same mass m leaves the pipe at end M in same time t, in which liquid will cover the distance $v_2 t$, therefore work done by liquid against the force due to pressure P_2 is

$$W_2 = P_2 A_2 v_2 t \qquad \ldots(2)$$

Net work done by pressure on the liquid in time t is,

$$W = W_1 - W_2 = P_1 A_1 v_1 t - P_2 A_2 v_2 t \qquad \ldots(3)$$

This work done on liquid by pressure increases its kinetic and potential energy.

Increase in kinetic energy of liquid is,

$$\Delta K = \frac{1}{2} m(v_2^2 - v_1^2) \qquad \ldots(4)$$

According to work-energy relation,

$$P_1 A_1 v_1 t - P_2 A_2 v_2 t = \frac{1}{2} m(v_2^2 - v_1^2) + mg(h_2 - h_1) \quad \ldots(6)$$

If there is no source and sink of liquid, then mass of liquid entering at end L is equal to the mass of liquid leaving the pipe at end M and is given by

$$A_1 v_1 \rho t = A_2 v_2 \rho t = m$$

or $$A_1 v_1 t = A_2 v_2 t = \frac{m}{\rho} \qquad \ldots(7)$$

From (6) and (7)

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} = \frac{1}{2} m(v_2^2 - v_1^2) + mg(h_2 - h_1)$$

or $$P_1 \frac{m}{\rho} + \frac{1}{2} m v_1^2 + mgh_1 = P_2 \frac{m}{\rho} + \frac{1}{2} m v_2^2 + mgh_2$$

or $$\frac{P}{\rho} + gh + \frac{1}{2} v^2 = \text{Constant}$$

Problem:- 5

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head $=$ pressure head + kinetic head + datum head

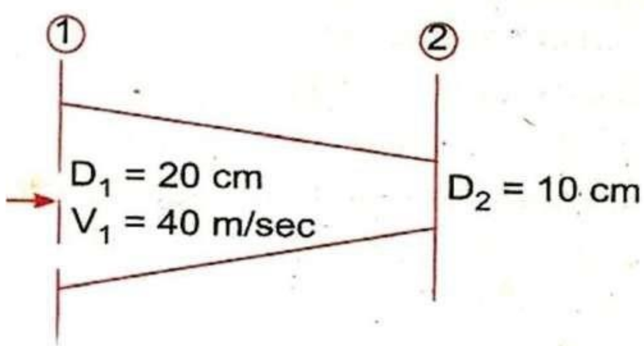
Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem:- 6

A pipe, through which water is flowing, is having diameters, 20cm and 10cm at the cross sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find thevelocity head at sections 1 and 2 and also rate of discharge.



Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area,
$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

$\therefore$
$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \textbf{0.815 m. Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$\therefore$
$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$\therefore$ Velocity head at section 2 $= \dfrac{V_2^2}{2g} = \dfrac{16.0 \times 16.0}{2 \times 9.81} = \textbf{83.047 m. Ans.}$

(*iii*) Rate of discharge $= A_1V_1 \quad \text{or} \quad A_2V_2$

$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$

Application of Bernoulli's equation:

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy consideration are involved.
It is also applied to following measuring devices

1. **Venturimeter**
2. **Pitot tube**

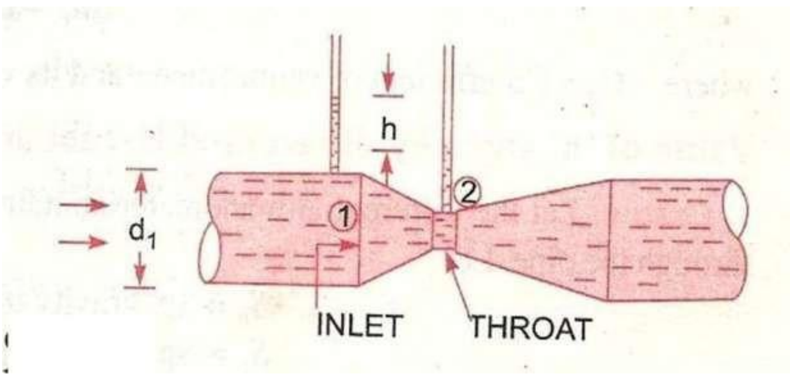
Venturimeter:

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe it consists of three parts.

I. **Short converging part**
II. **Throat**
III. **Diverging part**

Expression for rate of flow through venturimeter:

Consider a venturimeter is fitted in a horizontal pipe through which a fluid flowing



Let d_1 = diameter at inlet or at section (i)-(ii)

P_1 = pressure at section (1)-(1)

V_1 = velocity of fluid at section (1) – (1)

A_1= area at section (1) – (1) $= \frac{\pi}{4} d_1^2$

D_2, p_2 , v_2 , a_2 are corresponding values at section 2 applying Bernouli's equation at sections 1 and 2 we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h

So, $h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

Now applying continuity equation at sections 1 & 2 $a_1 v_1 = a_2 v_2$

Or $v_1 = \frac{a_2 v_2}{a_1}$

Substituting this value

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2{}^2 = 2gh\,\frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$$Q = a_2 v_2$$

$$= a_2\,\frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where Q = Theoretical discharge

Actual discharge will be less than theoretical discharge

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where C_d = co-efficient of venturimetre and value is less than 1

Value of 'h' given by differential U-tube manometer:

Case-i:

Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

Let S_h = Sp. Gravity of the heavier liquid

S_0 = Sp. Gravity of the liquid flowing through pipe

x = difference of the heavier liquid column in U-tube

$P_A - P_B = gx(\rho_g - \rho_0)$

$$\frac{P_A - P_B}{\rho_0 g} = x\left(\frac{\rho_g}{\rho_0} - 1\right)$$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case-ii

If the differential manometer contains a liquid lighter than the liquid flowing through the pipe

Where S_l = Specific gravity of lighter liquid in U-tube nanometre

So = Specific gravity of fluid flowing through in U-tube nanometre

x = Difference of lighter liquid columns in U- tube

The value of h is given by

$$h = x\left[1 - \frac{Sl}{S_0}\right]$$

Case-iii:

Inclined venturimetre with differential U-tube manometre

Let the differential manometer contains heavier liquid

Then h is given as

$$h = \left[\frac{P1}{\rho g} + z_1\right] - \left[\frac{P2}{\rho g} + z_2\right]$$

$$= x\left[\frac{Sh}{S_0} - 1\right]$$

Case-iv:

Similarly for inclined venturimetre in which differential manometer contaoins a liquid which is kighter than the liquid flowing through the pipe. Then

$$\mathbf{h} = \left[\frac{P1}{\rho g} + z_1\right] - \left[\frac{P2}{\rho g} + z_2\right]$$

$$\mathbf{h} = \mathbf{x}\left[1 - \frac{S}{S_0}\right]$$

Limitations:

- Bernoulli's equation has been derived underthe assumption that no external force except the gravity force is acting on the liquid. But in actual practice some external forces always acting on the liquid when effect the flow of liquid
- If the liquid is flowing in a curved path the energy due to centrifugal force should also be taken into account.

Pitot-tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

It is based on the principle that if the velocity flow at a point becomes zero, the pressure there is increased due to conversion of the kinetic energy into pressure energy.

The pitot-tube consists of a glass tube, bent an right angles

Consider two points 1 and 2 at te same level. Such a ay that 2 is at he inlet of pitot tube and one is the far away from the tube

Let P_1 = pressure at point 1

V_1 = velocity of fluid at point 1

P_2 = pressure at 2

V_2 = velocity of fluid at point 2

H = Depth of tube in the liquid

h = Rise of the liquid in the tube above the free surface

Applying Bernoulli's theorm

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\rho g} = H \qquad \frac{P_2}{\rho g} = (h + H)$$

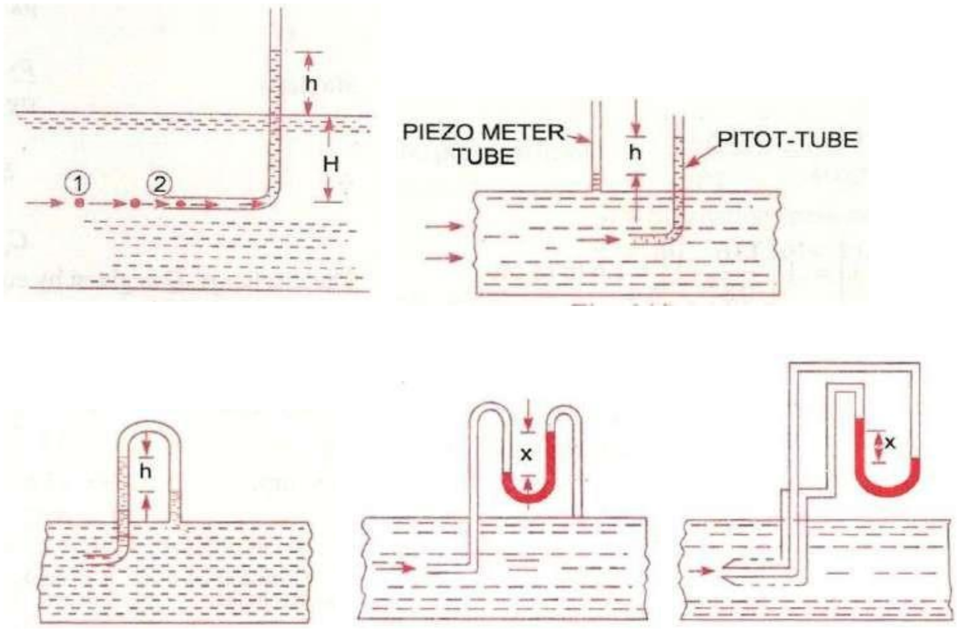
$$H + \frac{V_1^2}{2g} = h + H$$

$$V_1 = \sqrt{2gh}$$

Actual velocity, $V_{act} = C_v \sqrt{2gh}$

C_v = co-efficient of Pitot-tube

Different Arrangement of Pitot tubes



<u>**Numerical Problems:**</u>

Problem:- 7

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head $=$ pressure head $+$ kinetic head $+$ datum head

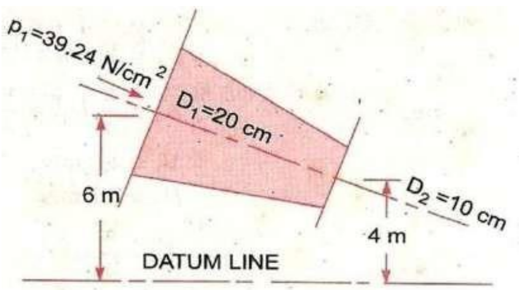
Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem:- 8

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35lit/s. The section 1 is 6m above datum and sedction 2 is 4m aboved datum. If the pressure at section 1 is 39.24 N/cm^2. Find the intensity of pressure at section 2



Solution:

Given

At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$

$p_1 = 39.24 \text{ N/cm}^2$
$= 39.24 \times 10^4 \text{ N/m}^2$

$z_1 = 6.0 \text{ m}$

At section 2, $D_2 = 0.10 \text{ m}$

$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$

$z_2 = 4 \text{ m}$

$p_2 = ?$

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$

and $V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

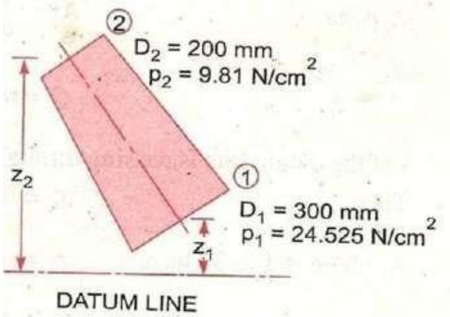
or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$
$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}.$$

Problem:- 9

Water is flowing through a pipe having diameter 300mm and 200 mm at the buttom and upper end respectively. The intensity of pressure at the bottom end is 9.81N/m^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s



Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

or $Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$

Now $A_1V_1 = A_2V_2 =$ rate of flow $= 0.04$

$\therefore$ $V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$

$\simeq 0.566 \text{ m/s}$

$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$

or $25 + .32 + z_1 = 10 + 1.623 + z_2$

or $25.32 + z_1 = 11.623 + z_2$

$\therefore$ $z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem:- 10

A horizontal venturimetre with inlet and throat diameters 10cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20cm of mercury. Determine the rate of flow. Take $C_d = 0.98$

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_0 = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \textbf{125.756 lit/s.}$$

Problem:- 11

An oil of Sp.gr. 0.8 is flowing through a horizontal venturimrtre having inlet diameter 20cm and throaty diameter 10 cm. The oil mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimetre. Take Cd = 0.98

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3\text{/s}$$

$= 70465 \text{ cm}^3\text{/s} =$ **70.465 litres/s. Ans.**

Problem:- 12

A horizontal venturimrtre with inlet and throat diameters 20cm and 10 cm respectively is used to measure the flow of oil of Sp. gr. The discharge of oil through venturimetre is 60lit/s . Find thereading of oil-mercury differential manometer. Take C_d = 0.98

Solution. Given : $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3\text{/s}$

Using the equation (6.8), $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h}$

$= \frac{1071068.78\sqrt{h}}{304}$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore$ $$h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$\therefore$ $$289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$\therefore$ $$x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm.**

Problem:-13

A static pitot-tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and is perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60mm of water. Take C_v = 0.98

Solution. Given :

Dia. of pipe, d = 300 mm = 0.30 m

Diff. of pressure head, h = 60 mm of water = .06 m of water

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80 \times$ Central velocity

Central velocity is given by equation (6.14)

$$= C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore$ $$\overline{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$$

Discharge, $$Q = \text{Area of pipe} \times \overline{V}$$

$$= \frac{\pi}{4}d^2 \times \overline{V} = \frac{\pi}{4}(.30)^2 \times 0.8504 = \mathbf{0.06 \ m^3/s.\ Ans.}$$

FM — 5th Chapter
(Orifices, notches and weirs)

Orifices –

It is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing.

A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank containing the fluid.

Orifice as well as mouthpieces are used for measuring the rate of flow of fluid.

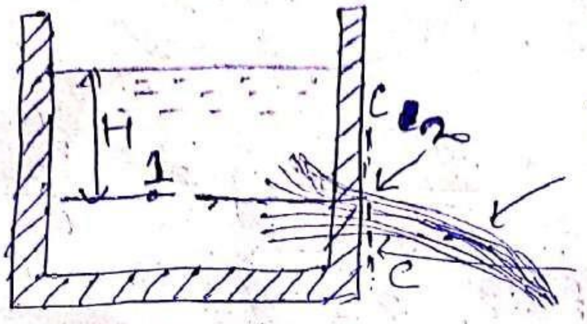
Classification of orificies :

- **Small orifice** (If the head of liquid from the centre of orifice is more than five times the depth of orifice)
- **large orifice** (If the head of liquid from the centre of orifice is less than five times the depth of orifice)

The orifices are classified as (i) Circular orifice (ii) triangular orifice (iii) Rectangular orifice (iv) Square orifice. (according to the cross-sectional area)

Flow through an orifice :

Consider a tank fitted with a circular orifice in one of its sides.



Let H is the head of liquid above the centre of the orifice.
The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice.
The area of jet of fluid decreases at section (C-C) called vena-contracta, which is at a distance of half of diameter of the orifice.
At this section the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice.
Beyond this section, the jet diverges and is attracted in downward direction by the gravity.
Consider two points 1 & 2. ~~which~~ point 1 is inside the tank and point 2 is at vena-contracta.
Let the flow is steady and at a constant head H.

Applying Bernoulli's equation at point 1 & 2.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\therefore \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $\frac{P_1}{\rho g} = H$, $\frac{P_2}{\rho g} = 0$ (atmospheric pressure)

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$$\therefore H + 0 = 0 + \frac{v_2^2}{2g}$$

$$\Rightarrow v_2 = \sqrt{2gH} \quad \text{(Theoritical velocity)}$$

Actual velocity is always less than the theoritical velocity.

Hydraulic Co-efficients –

The hydraulic co-efficients are :

1. Co-efficient of velocity (C_v)
2. Co-efficient of contraction (C_c)
3. Co-efficient of discharge (C_d)

Co-efficient of velocity :

It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet.

Mathematically –

$$C_v = \frac{V}{\sqrt{2gH}}$$

where V = Actual velocity

$\sqrt{2gH}$ = Theoretical velocity

The value of C_v varies from 0.95 to 0.99.
The general value of C_v is 0.98.

Co-efficient of Contraction –

It is defines as the ratio of the area of the jet at vena-contracta to the area of the orifice.

$$C_c = \frac{a_c}{a}$$

The value of C_c varies 0.61 to 0.69.
The general value of C_c is 0.64.

Co-efficient of Discharge :-

It is defined as the ratio of the actual discharge from an orifice to the theoritical discharge from the orifice.

Mathematically

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual Velocity} \times \text{Actual Area}}{\text{Theoritical velocity} \times \text{Theoritical Area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoritical velocity}} \times \frac{\text{Actual Area}}{\text{Theoritical area}}$$

$$\boxed{C_d = C_v \times C_c}$$

The value of C_d varies from 0.61 to 0.65.

The general value of C_d 0.62

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Notch - It is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

Weir - It is a concrete or masonary structure, placed in an open channel over which the flow occurs.
It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel.

☆ The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

Classification of Notches and Weirs :-

The notches are classified as :

① According to the shape of the opening
  (a) Rectangular notch
  (b) Triangular notch
  (c) Trapezoidal notch
  (d) Stepped notch

② According to the effect of the sides on the nappe :
  (a) Notch with end contraction
  (b) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening, the shape of the crest.

① According to the shape of the opening:

(a) Rectangular weir

(b) Triangular weir

(c) Trapezoidal weir

② According to the shape of crest:

(a) Sharp-crested weir

(b) Broad-crested weir

(c) Narrow-crested weir

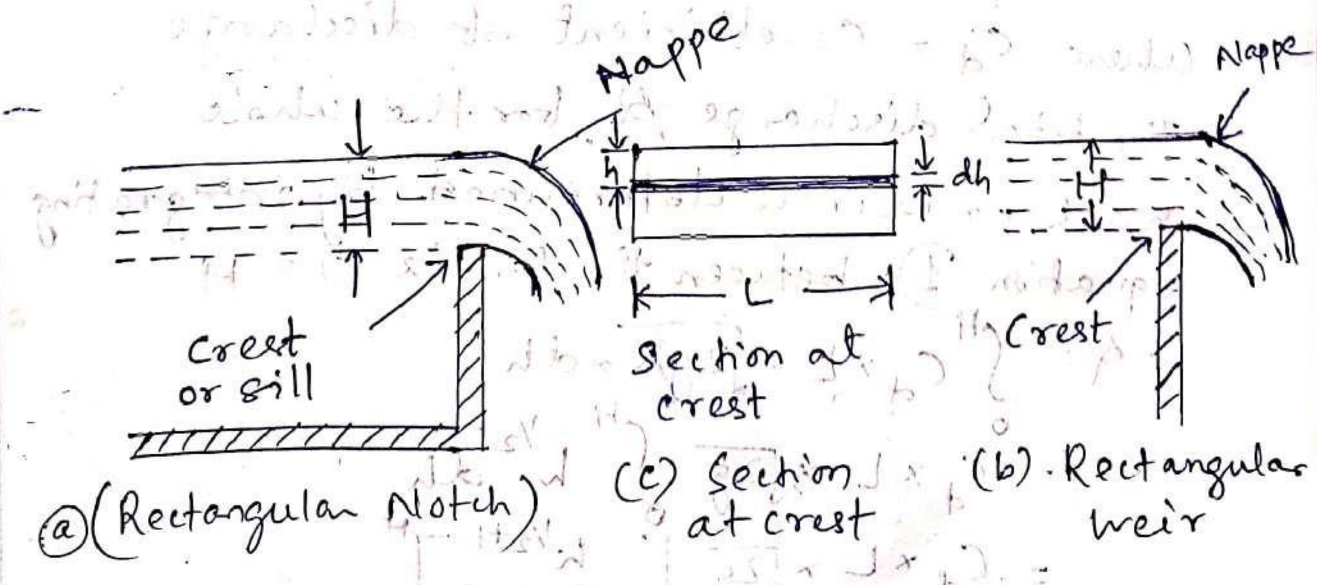
(d) Ogee-shaped weir

③ According to the effect of sides on the emerging nappe:

(a) Weir with end contraction

(b) Weir without end contraction.

Discharge over a Rectangular notch or weir:



Consider a rectangular notch or weir provided in a channel carrying water.

Let, H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

The area of strip $= L \times dh$

The theoritical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is,

$dQ = C_d \times$ Area of Strip $\times$ Theoritical velocity

$$= C_d \times L \times dh \times \sqrt{2gh} \quad \text{——— (1)}$$

where C_d = Co-efficient of discharge

∴ The total discharge, Q, for the whole notch or weir is determined by integrating equation (1) between the limits 0 & H

$$\therefore Q = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{1/2+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H = \boxed{\frac{2}{3} C_d \times L \times \sqrt{2g}\, [H]^{3/2}}$$

Problem on Rectangular notch –

1) Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$

Solution :

Given data :

Length of the notch = $L = 2.0$ m

Head over notch = $H = 300$ mm $= 0.3$ m

$C_d = 0.60$

$$\text{Discharge, } Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\,[H]^{3/2}$$

$$= \frac{2}{3} \times 0.60 \times 2 \times \sqrt{2 \times 9.81} \times [0.3]^{3/2}$$

$$= 0.582 \text{ m}^3/\text{sec}.$$

Problem on Rectangular Weir :

1) Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 lit/sec. Take $C_d = 0.6$ and neglect end contractions.

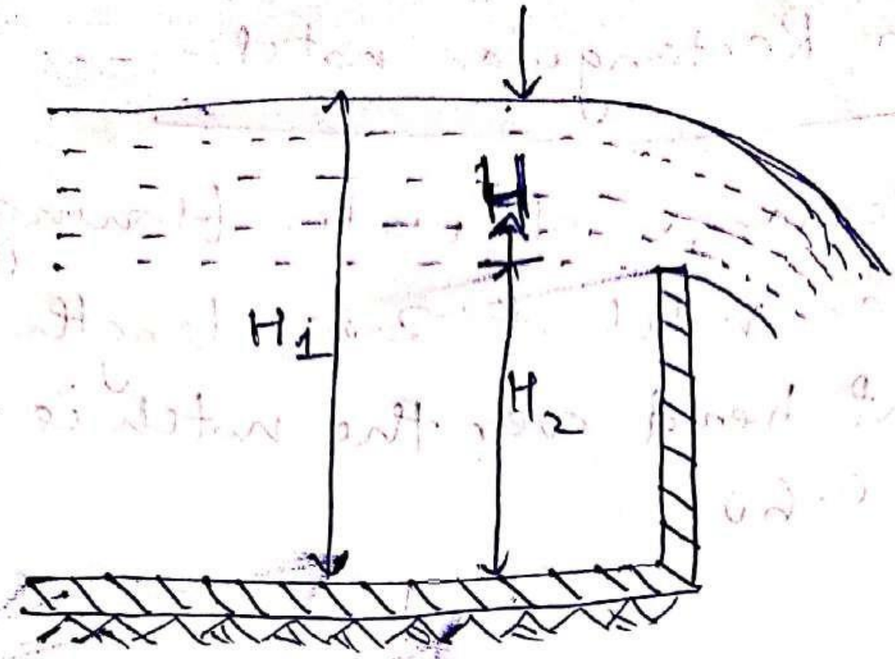
Solution :

Given data :

Length of weir, $L = 6$ m $\quad C_d = 0.60$

Depth of weir, $H_1 = 1.8$ m

Discharge $Q = 2000$ lit/sec $= 2$ m^3/sec



Let H is the height of water above the crest of water and H_2 = height of weir.

The discharge over the weir, Q =

$$\frac{2}{3} C_d \times L \times \sqrt{2g}\, H^{3/2}$$

$$\Rightarrow 2 = \frac{2}{3} \times 0.60 \times 6 \times \sqrt{2 \times g} \times H^{3/2}$$

$$\Rightarrow 2 = \frac{2}{3} \times 0.60 \times 6 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$\Rightarrow H^{3/2} = \frac{2}{10.623}$$

$$\Rightarrow H = \left(\frac{2}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

∴ Height of water, $H_2 = H_1 - H$

$$= 1.8 - 0.328 = 1.472 \text{ m}$$

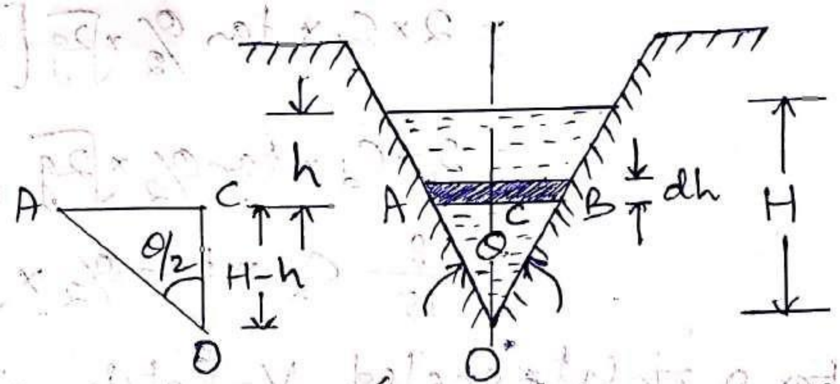
Discharge over a triangular notch or weir :

The expression for the discharge over a triangular notch is derived as:

Let, H = head of water above the V-notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water.



(Triangular Notch)

From the figure,

$$\tan\theta/2 = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$\therefore AC = (H-h)\tan\theta/2$$

Width of strip $= AB = 2AC = 2(H-h)\tan\theta/2$

$\therefore$ Area of strip $= 2(H-h)\tan\theta/2 \times dh$

The theoritical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge (dQ) through the strip =

$$dQ = C_d \times \text{Area of strip} \times V_{th}$$

$$= C_d \times 2(H-h)\tan\theta/2 \times dh \times \sqrt{2gh}$$

$$= 2C_d(H-h)\tan\theta/2 \times \sqrt{2gh} \times dh$$

$\therefore$ Total Discharge, $Q = \int_0^H 2 C_d (H-h) \tan\theta/2 \times \sqrt{2gh} \times dh$

$= 2 C_d \times \tan\theta/2 \times \sqrt{2g} \int_0^H (H-h) h^{1/2} dh$

$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g} \int_0^H (Hh^{1/2} - h^{3/2}) dh$

$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g} \left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H$

$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g} \left[\frac{2}{3} H.H^{3/2} - \frac{2}{5} H^{5/2} \right]$

$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g} \left[2/3 H^{5/2} - \frac{2}{5} H^{5/2} \right]$

$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right]$

$= \frac{8}{15} C_d \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}$ —— (1)

For a right-angled V-notch, if $C_d = 0.60$

$\theta = 90°$, $\tan\theta/2 = 1$

Discharge $Q = \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$ —— (2)

$$\boxed{Q = 1.417 H^{5/2}}$$

Problem on Triangular notch –

1) Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.60$.

Solution - Given data :

Angle of V-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.60$

Discharge, Q, over a V-notch =

$$Q = \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan\frac{60^\circ}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.818 \times 0.049 = 0.040 \ m^3/sec.$$

2) A rectangular channel 2.0 m wide has a discharge of 250 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not to exceed 1.3 m. Take $C_d = 0.62$

Solution - Given data :

Width of rectangular channel, $L = 2m$; $C_d = 0.62$

$\theta = 90^\circ$ (Right angled V-notch) $Q = 250$ lit/sec $= 0.25 \ m^3/sec$

Depth of water in channel = 1.3 m

Let, the height of water over V-notch = H

The rate of flow or discharge through V-notch = $Q = \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\theta/2 \times H^{5/2}$

$$\Rightarrow 0.25 = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times \tan\frac{90^\circ}{2} \times H^{5/2}$$

$$\Rightarrow 0.25 = \frac{8}{15} \times 0.62 \times 4.429 \times 1 \times H^{5/2}$$

$$\Rightarrow H^{5/2} = \frac{0.25 \times 15}{8 \times 0.62 \times 4.429} = 0.17$$

$$\Rightarrow H = (0.17)^{2/5} = (0.17)^{0.4} = 0.493 \ m$$

∴ Position of apex of the notch from the bed of channel

= (depth of water in channel) − (height of water over V-notch)

$= 1.3 - 0.493 = 0.807$ m

6th Chapter (Flow through Pipes)

Definition of Pipe :-

It is a hollow cylinder of metal, wood or other material used for the conveyance of water, gas, steam, petroleum etc.

Loss of energy in pipes :

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

Energy Losses

1. Major Energy Losses
   ↓
   This is due to friction and it is calculated by the following formulae:
   (a) Darcy-Weisbach formula
   (b) Chezy's formula

2. Minor Energy losses
   ↓
   This is due to
   (a) Sudden expansion of pipe.
   (b) Sudden contraction of pipe
   (c) Bend in pipe
   (d) Pipe fittings etc.
   (e) An obstruction in pipe.

Loss of energy (or head) due to friction :

(a) Darcy-Weisbach formula :

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation →

$$h_f = \frac{4fLV^2}{d \times 2g}$$

Where h_f = loss of head due to friction.

f = Co-efficient of friction which is a function of Reynolds number

$= \frac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \frac{0.079}{R_e^{1/4}}$ for R_e (varying from 4000 to 10^6)

L = Length of pipe

V = Mean velocity of flow

d = diameter of pipe.

(b) Chezy's formula for loss of head due to friction in pipes. :—

for loss of head due to friction in pipes =

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{——— (1)}$$

Where h_f = loss of head due to friction

A = Area of cross-section of pipe

V = Mean velocity of flow

P = Wetted perimeter of pipe

L = Length of pipe.

The ratio of $\frac{A}{P} \left(\frac{\text{Area of flow}}{\text{Wetted Perimeter}} \right) = m$

is called hydraulic mean depth or

hydraulic radius.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d}$

$\therefore$

$$\boxed{m = \frac{d}{4}}$$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (1), we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m}$$

$$\Rightarrow V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\Rightarrow V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}}$$

$$\Rightarrow V = \sqrt{\frac{\rho g}{f'}}\sqrt{m\frac{h_f}{L}} \quad \text{——— (2)}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is Chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the value of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation (2), we get

$$\boxed{V = C\sqrt{mi}} \quad \text{——— (3)}$$

The equation (3) is known as Chezy's formula.

Problems on Darcy formula and Chezy's formula :-

1) Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m through which water is flowing at a velocity of 3 m/sec using (i) Darcy formula (ii) Chezy's formula for which C = 60. Take ν for water = 0.01 stoke.

Solution – Given data :

Diameter of pipe, d = 300 mm = 0.30 m

Length of pipe, L = 50 m.

Velocity of flow V = 3 m/sec

Chezy's constant C = 60

Kinematic viscosity ν = 0.01 stoke

$= 0.01\ cm^2/sec$

$= 0.01 \times 10^{-4}\ m^2/sec$

(i) Darcy formula is $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$

where 'f' = co-efficient of friction is a function of Reynolds number, Re

$$R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5$$

$$\therefore f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

$\therefore$ Head lost, $h_f = \dfrac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81}$

$= 0.782$ m

(ii) Chezy's formula :

$$V = C\sqrt{mi}$$

Where $C = 60$, $m = \dfrac{d}{4} = \dfrac{0.30}{4} = 0.075$ m.

$V = 3$ m/sec, $L = 50$ m

$\therefore \quad 3 = 60\sqrt{0.075 \times i}$

$\Rightarrow \boxed{i = \left(\dfrac{3}{60}\right)^2 \times \dfrac{1}{0.075} = 0.033}$

But $\boxed{i = \dfrac{h_f}{L} = \dfrac{h_f}{50}}$

Equating the two values of i, we get

$0.033 = \dfrac{h_f}{50}$

$\Rightarrow h_f = 50 \times 0.033 = 1.665$ m.

2) Find the diameter of pipe of length 2000 m when the rate of flow of water through the pipe is 200 lit/sec and the head lost due to friction is 4 m. Take the value of $C = 50$ in Chezy's formula.

Solution :

Given data :

Length of pipe, $L = 2000$ m.

Discharge, $Q = 200$ lit/sec
$= 0.2\ m^3/sec$

Head lost due to friction, $h_f = 4$ m.

Value of Chezy's Constant, $C = 50$

Let the diameter of pipe $= d$

Velocity of flow, $V = \frac{\text{Discharge}}{\text{Area}}$

$$= \frac{Q}{\frac{\pi}{4} d^2} = \frac{0.2}{\frac{\pi}{4} d^2}$$

$$= \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $m = \frac{d}{4}$

Loss of head per unit length,

$$i = \frac{h_f}{L} = \frac{4}{2000} = 0.002$$

Chezy's formula, $V = C\sqrt{m i}$

Substituting the values of V, m, i and C, we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

$$\Rightarrow \sqrt{\frac{d}{4} \times 0.002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{0.005}{d^2}$$

Squaring both sides, $\frac{d}{4} \times 0.002 = \frac{0.005^2}{d^4}$

$$\Rightarrow \frac{d}{4} \times 0.002 = \frac{0.000025}{d^4}$$

$$\Rightarrow d^5 = \frac{4 \times 0.000025}{0.002} = 0.05$$

$$\Rightarrow d = \sqrt[5]{0.05} = 0.55\ m.$$

Hydraulic gradient and Total Energy line :-

It is very useful in the study of flow of fluids through pipes.

Hydraulic Gradient Line (HGL) :-

It is defined as the line which gives the sum of pressure head (P/w) and datum head (Z) of a flowing fluid in a pipe with respect to some reference line.

Total Energy Line (TEL) :-

It is defined as the line which gives the sum of pressure head ($\frac{P}{w}$), datum head (Z) and kinetic head ($\frac{V^2}{2g}$) of a flowing fluid in a pipe with respect to some reference line.

It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.

7th Chapter (Impact of Jets)

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure.

The impact of jet means the force exerted by the jet on a plate which is stationary or moving.

This force is obtained from Newton's second law of motion or from Impulse-momentum equation.

1. The force exerted by the jet on a plate, ~~when~~ (stationary plate), when
   (a) Plate is vertical to the jet
   (b) Plate is inclined to the jet
   (c) Plate is curved.
2. The force exerted by the jet on a moving plate, when
   (a) Plate is vertical to the jet
   (b) Plate is inclined to the jet
   (c) plate is curved.

Force exerted by the jet on a stationary (fixed) vertical Plate : —

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let, V = velocity of the jet

d = diameter of the jet

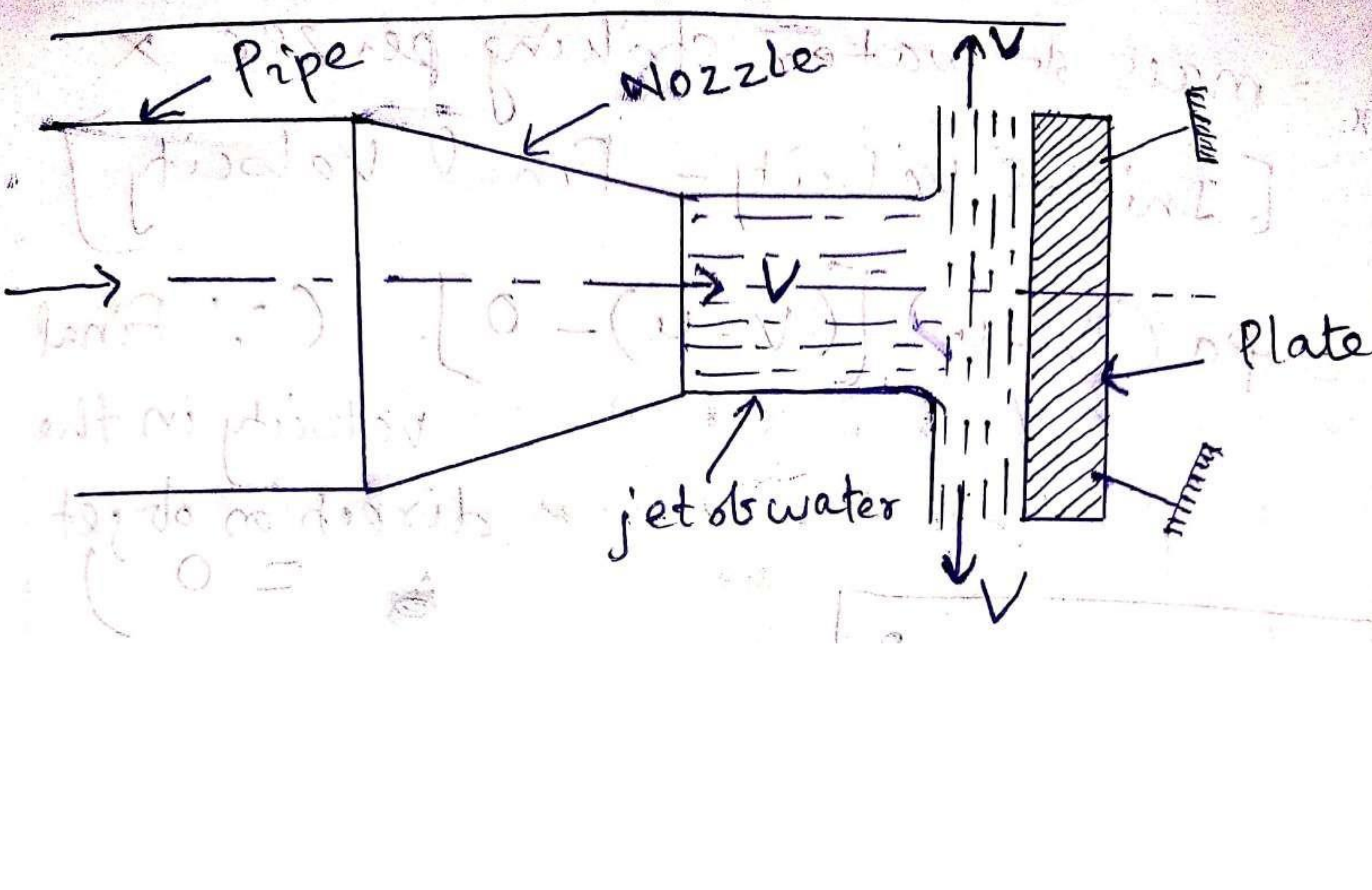
a = area of cross-section of the jet $= \frac{\pi}{4} d^2$

The jet after striking the plate, move along the plate.

But the plate is at right angles to the jet.

Hence the jet after striking, deflects through $90°$.

So the component of the velocity of jet in the direction of jet, after striking is zero.


Pipe
Nozzle
V
V
Plate
jet of water
V

∴ The force exerted by the jet on the plate in the direction of jet,

F_x = Rate of change of momentum in the direction of force

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{time}}$$

$$= \frac{(\text{mass} \times \text{Initial Velocity} - \text{mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{mass}}{\text{time}}[\text{Initial velocity} - \text{Final velocity}]$$

= (mass/sec) × (velocity of jet before striking − velocity of jet after striking)

$= \rho a V[V - 0]$ ($\because$ mass/sec $= \rho \times aV$)

$$\boxed{F_x = \rho a V^2}$$

☆ If the force exerted by the jet on the plate is calculated, then

~~F_x = m Initial~~ $F_x = \frac{\text{mass}}{\text{time}}$ (Initial velocity − Final velocity)

If the force exerted on the jet is calculated, then

$$F_x = \frac{\text{mass}}{\text{time}}(\text{Final velocity} - \text{Initial velocity})$$

Force exerted by a jet on moving vertical flat plate:

The jet of water strikes a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity ~~of the jet~~ of the jet

a = Area of cross-section of the jet

u = Velocity of the flat plate.

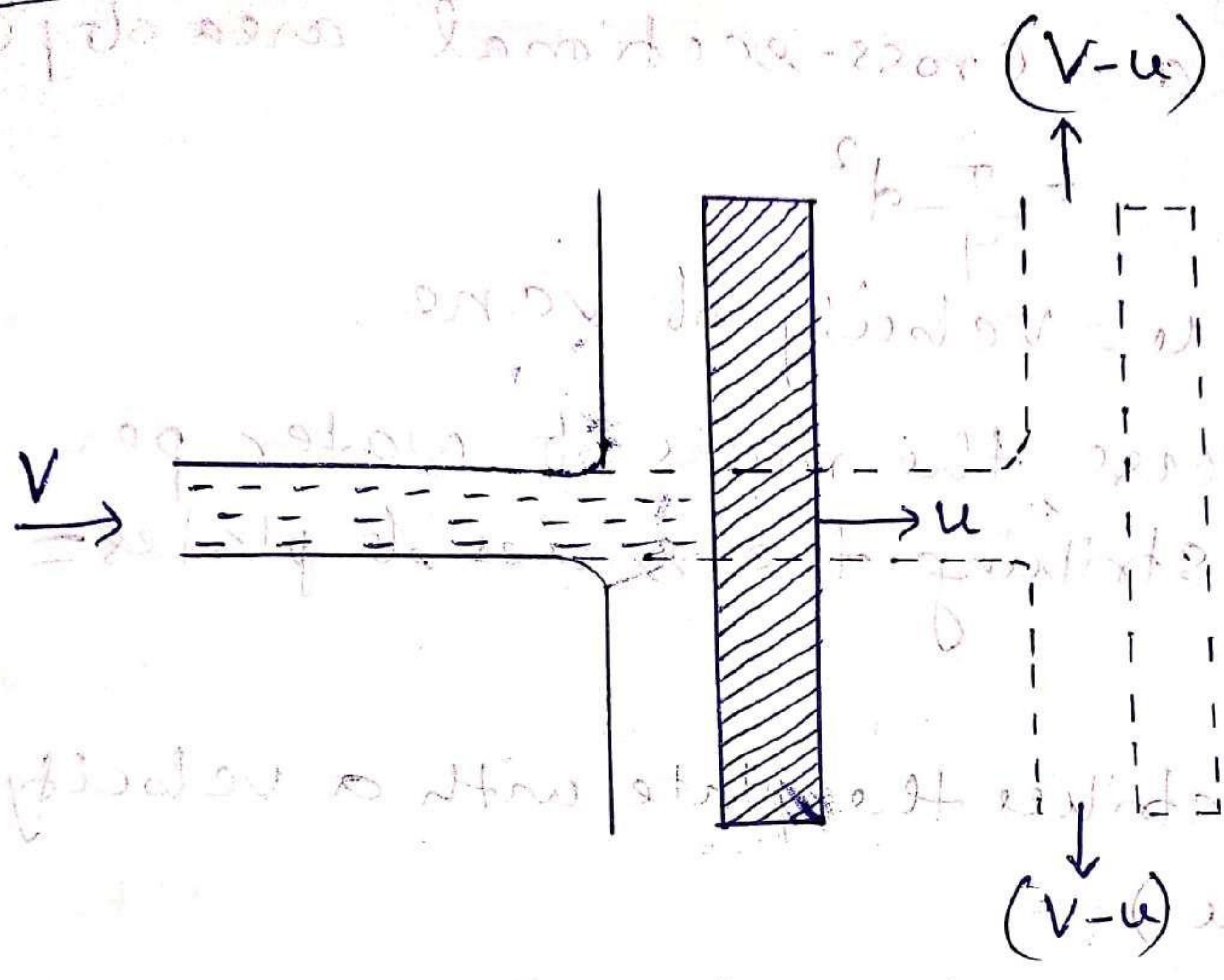
In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity which is equal to the absolute velocity of jet of water minus the velocity of plate.

$\therefore$ the relative velocity of the jet with respect to plate $= V-u$

Mass of water striking the plate per sec

$= \rho \times$ Area of jet $\times$ Velocity with which jet strikes the plate

$= \rho a \times [V-u]$



(V-u)
V
u
(V-u)

∴ Force exerted by the jet on the moving plate in the direction of jet,

F_x = mass of water striking per sec × [Initial velocity − Final velocity]

$= \rho a(V-u)\,[(V-u)-0]$ (∵ Final velocity in the direction of jet = 0)

$$\boxed{F_x = \rho a (V-u)^2} \quad \text{——— (1)}$$

In this case, the work done per second by the jet on the moving plate

= Force × $\frac{\text{Distance in the direction of force}}{\text{time}}$

$= F_x \times u$

$$\boxed{W = \rho a (V-u)^2 \times u} \quad \text{——— (2)}$$

~~from the~~ In the equation (2),
the value of $\rho = 1000\ kg/m^3$ for water

The unit of W is Nm/sec.

Force exerted by a jet of water on series of vanes:-

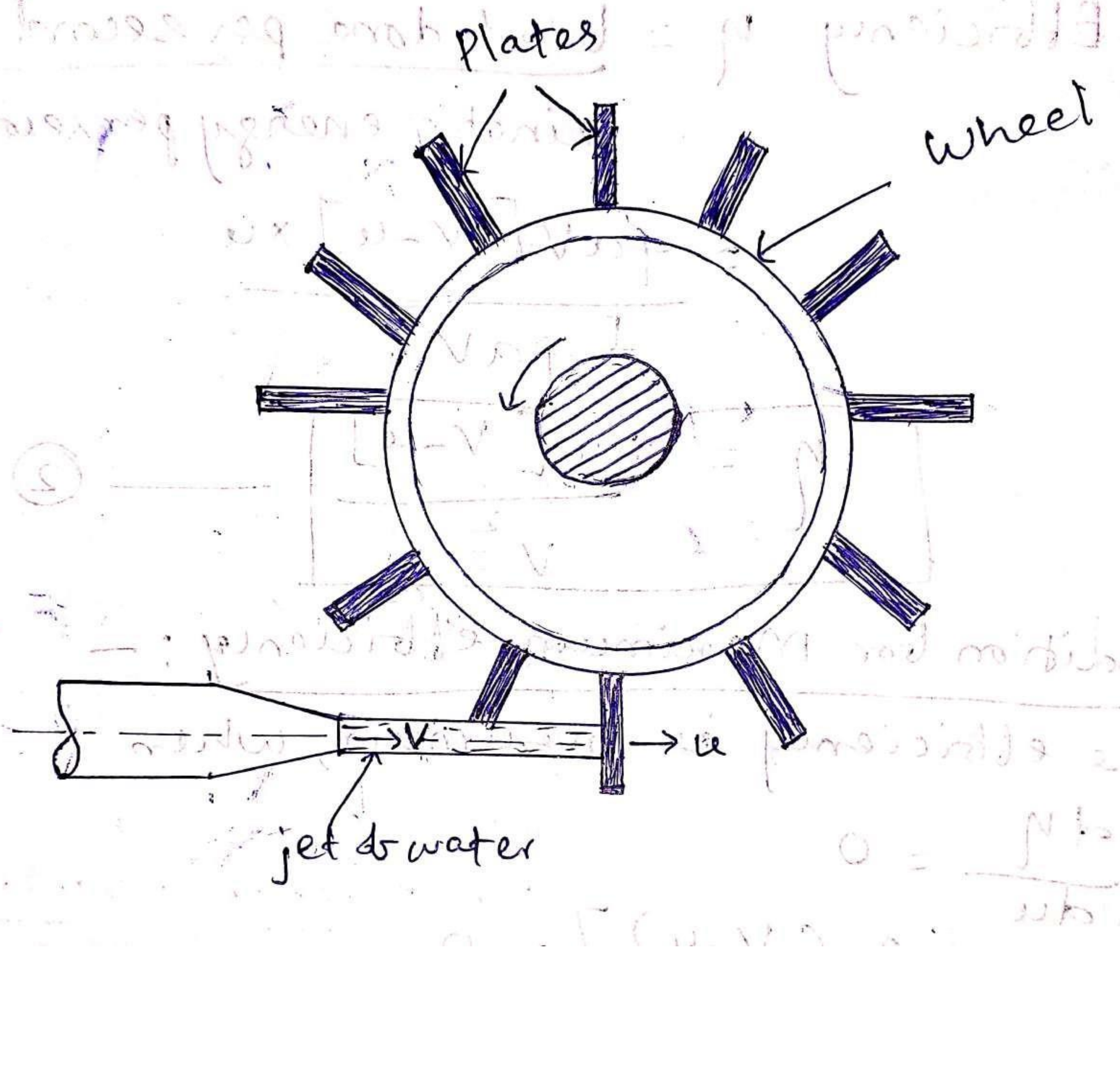
In this case, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart.

The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving.

The 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate.

So each plate appears before the jet successively and the jet exerts force on each plate.

The wheel starts moving at a constant speed.



Plates
wheel
V
u
jet of water

Force exerted on a series of Radial Curved Vanes :-

Consider a series of radial curved vanes mounted on a wheel.

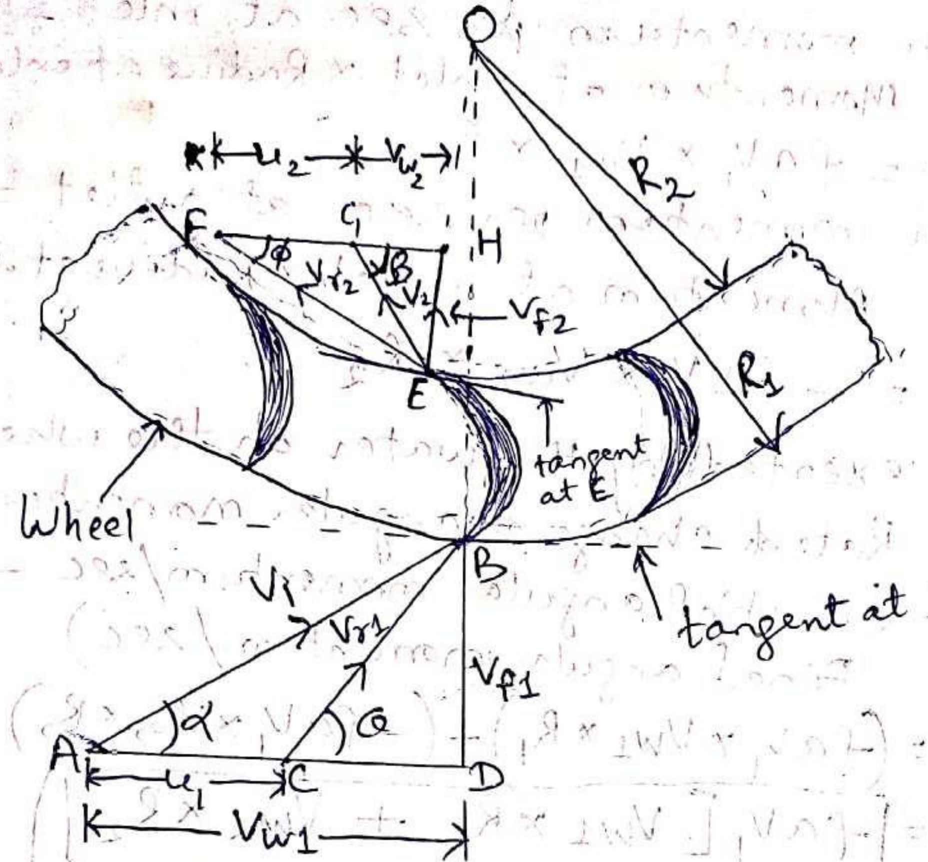
The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

Let R_1 = Radius of wheel at inlet of the vane.

R_2 = Radius of wheel at the outlet of the vane.

ω = Angular speed of the wheel

$\therefore \quad u_1 = \omega R_1 \quad$ and $\quad u_2 = \omega R_2$



From the velocity triangles at inlet and outlet,

The mass of water striking per sec. for a series of vanes = Mass of water coming out from nozzle per sec. $= \rho a V_1$

where a = Area of jet, V_1 = Velocity of jet.

Momentum of water striking the vanes in the tangential direction per sec. at inlet =

= Mass of water per sec. × Component of V_1 in the tangential direction

$= \boxed{\rho a V_1 \times V_{w1}}$ $\quad (\because V_1 \cos\alpha = V_{w1})$

Momentum of water at outlet per sec.

$= \rho a V_1 \times$ Component of V_2 in the tangential direction

$= \rho a V_1 \times (-V_2 \cos\beta) = \boxed{-\rho a V_1 \times V_{w2}}$

$(\because V_2 \cos\beta = V_{w2})$

-ve sign indicates the velocity V_2 at outlet is in opposite direction.

Angular momentum per sec. at inlet = Momentum at inlet × Radius at inlet

$= \rho a V_1 \times V_{w1} \times R_1$

Angular momentum per sec. at outlet = Momentum at outlet × Radius at outlet

$= -\rho a V_1 \times V_{w2} \times R_2$

Torque exerted by the water on the wheel =

T = Rate of change of angular momentum

= (Initial angular momentum/sec − Final angular momentum / sec)

$= (\rho a V_1 \times V_{w1} \times R_1) - (-\rho a V_1 \times V_{w2} \times R_2)$

$$= \rho a V_1 [V_{w1} \times R_1 + V_{w2} \times R_2]$$

Workdone per sec. on the wheel = Torque × Angular velocity = $T \times \omega$

$= \rho a V_1 [V_{w1} \times R_1 + V_{w2} \times R_2] \times \omega$

$= \rho a V_1 [V_{w1} \times R_1 \times \omega + V_{w2} \times R_2 \times \omega]$

$= \rho a V_1 [V_{w1} u_1 + V_{w2} u_2]$ $(\because u_1 = \omega R_1$ and $u_2 = \omega R_2)$

If the angle β is an obtuse angle, then workdone per sec. $= \rho a V_1 [V_{w1} u_1 - V_{w2} u_2]$

$\therefore$ The workdone per sec. on the wheel =

$$\rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]$$

If the discharge is radial at outlet, then $\beta = 90°$ and the workdone $= \rho a V_1 [V_{w1} u_1]$

$(\because V_{w2} = 0)$

Efficiency of the Radial Curved Vane :-

The workdone per sec. on the wheel is the output of the system.
The Initial kinetic energy per sec of the jet is input.

$$\therefore \text{Efficiency}, \eta = \frac{\text{workdone per sec.}}{\text{Kinetic energy per sec.}}$$

$$= \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2}(m/sec) \times V_1^2}$$

$$= \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2} \rho a V_1 \times V_1^2}$$

$$= \boxed{\frac{2 [V_{w_1} u_1 \pm V_{w_2} u_2]}{V_1^2}}$$

Workdone per sec. on the wheel = Change in K.E per sec. of the ~~wheel~~ jet.

$= (\text{Initial K.E per sec.} - \text{Final K.E per sec.})$

$= \left(\frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2\right)$

$= \frac{1}{2} m (V_1^2 - V_2^2) = \boxed{\frac{1}{2}(\rho a V_1^2)(V_1^2 - V_2^2)}$

$(\because m/sec = \rho a V_1)$

$$\therefore \text{Efficiency}, \eta = \frac{\text{workdone per sec. on the wheel}}{\text{Initial K.E per sec. of the jet}}$$

$$= \frac{\frac{1}{2} \rho a V_1^2 (V_1^2 - V_2^2)}{\frac{1}{2}(\rho a V_1^2) . V_1^2}$$

$$= \frac{V_1^2 - V_2^2}{V_1^2} = \boxed{1 - \frac{V_2^2}{V_1^2}}$$

From the above equation, the efficiency is maximum when V_2 is minimum.