Lecture notes on

STRUCTURAL MECHANICS

(TH-1, 3^{RD} SEM)

CIVIL ENGINEERING

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Chapter 1

SIMPLE AND COMPLEX STRESS AND STRAIN .

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SIMPLE STRESS AND STRAIN :

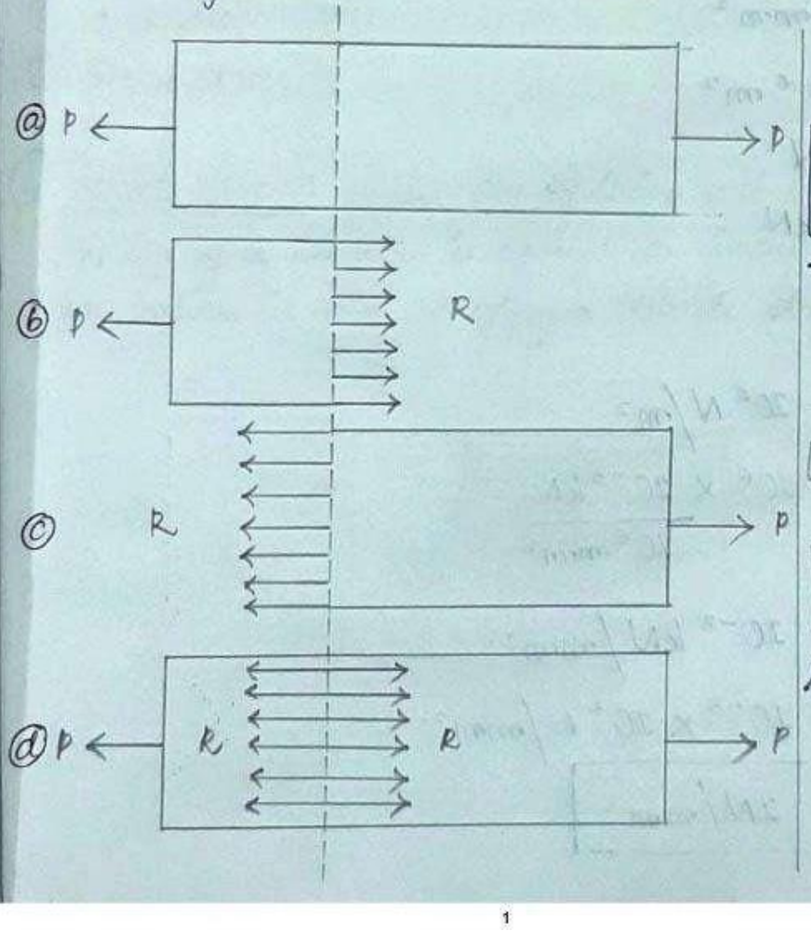
* **Stress**: A resistance force offered by a body against the deformation is called stress.
(change in shape and size)

② The external force acting on the body is called load.

③ In a single line, load is applied on the body and stress is induced in the material of the body.

④ Stress is denoted by 'σ' (sigma).

⑤ Let a rod of uniform cross-sectional area is subjected to pulling force (P), to resist the deformation, then an internal force (R), will be induced inside the material.



$\therefore$ Stress $= \frac{R}{A}$

$$\boxed{\sigma = \frac{P}{A}} \quad [\because R = P]$$

→ Unit : $\frac{\text{Newton}}{(\text{metre})^2}$

$= \frac{N}{m^2}$ = Pascal

$R = P$ no tearing of the body

[applied force = resisting force]

Stress = $\frac{\text{Internal resisting force}}{\text{area}}$

UNITS OF STRESS

kilo = 10^3
Mega = 10^6
Giga = 10^9
Tera = 10^{12}
Milli = 10^{-3}
Micro = 10^{-6}
Nano = 10^{-9}
Pico = 10^{-12}

1 Pascal = $1\ N/m^2$.

1 KiloPascal = $10^3\ N/m^2$.

1 Mega Pascal = $10^6\ N/m^2$.

1 Giga Pascal = $10^9\ N/m^2$

* $1\ m \rightarrow 10^3\ mm$

  $1\ mm \rightarrow \frac{1}{10^3}\ m = 10^{-3}\ m$

* $1\ m^2 = (10^3)^2\ mm^2$.

  $= 10^6\ mm^2$.

* $1\ mm^2 = 10^{-6}\ m^2$.
* $1\ kN = 10^3\ N$
* $1\ N = 10^{-3}\ kN$

We know,

$$1\ MPa = 10^6\ N/m^2$$

$$= 10^6 \times \frac{10^{-3}\ kN}{10^6\ mm^2}$$

$$= 10^{-3}\ kN/mm^2.$$

$$= 10^{-3} \times 10^3\ N/mm^2.$$

$$\boxed{1\ MPa = 1\ N/mm^2.}$$

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* **Strain:** This is the ratio of change in dimension to the original dimension.

→ It is denoted by 'e'.

⇒ $e = \frac{\Delta l}{l}$ ⇒ $\frac{\text{change in length}}{\text{original length}}$ ⇒ $\frac{\Delta d}{d}$ ⇒ $\frac{\Delta v}{v}$

linear strain [→ tensile, → compressive]; lateral strain; volumetric strain

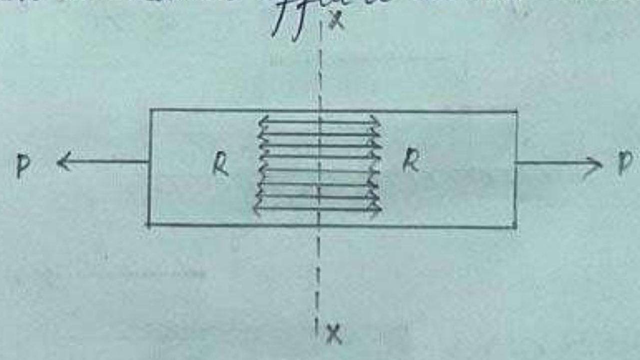
→ Strain has no units i.e. it is unitless.

* **Types of Stress:**

A material is capable of offering three types of stresses:

(i) Tensile stress
(ii) Compressive stress
(iii) Shear stress

(i) **Tensile Stress:** When the offering resistance by a section of a member is against an increase in length, the section is said to offer a tensile stress.



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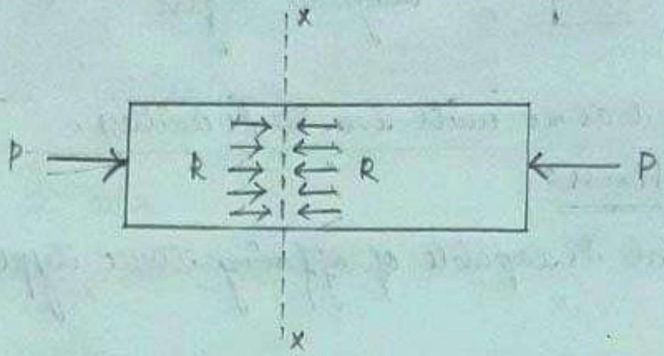
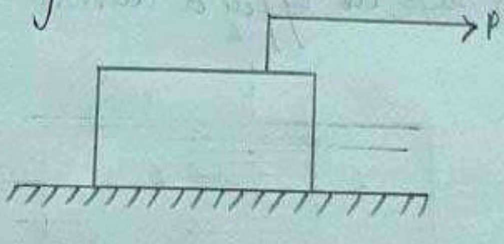
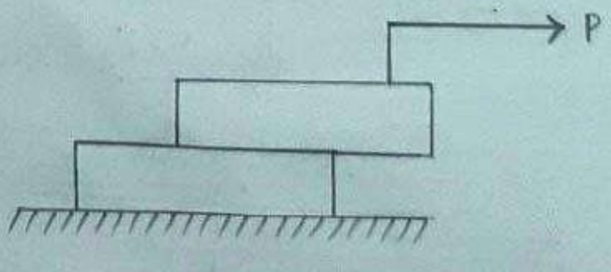
(ii) **Compressive Stress:** When the offering resistance by a section of a member is against a decrease in length, the section is said to offer a compressive stress.



(iii) **Shear Stress:** (τ) A load P is applied tangentially along the top of the face of the body of which, bottom face is fixed. Such force acting tangentially along a surface is called as shear force.





→ The resistance provided in this case is called shear resistance.

→ Shear stress is defined as the ratio of shear resistance to the shear area.

mathematically, $\tau = \frac{R}{A}$

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* Types of Strain:

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Strain can be of 4 types:

(i) **Tensile strain:** The ratio of increase in length to its original length, is called tensile strain.

$$e = \frac{\Delta l}{l}$$

→ here, Δl → change in increase in length

(ii) **Compressive Strain:** The ratio of decrease in length to the original length, is called compressive strain.

$$e = \frac{\Delta l}{l}$$

→ here, Δl → change in decrease in ~~leg~~ length

(iii) **Lateral strain:** The ratio of the change in lateral dimension to the original lateral dimension, is called lateral strain.

$$e = \frac{\Delta d}{d}$$

→ here, Δd → change in lateral dimension

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(iv) <u>Volumetric strain</u>: The ratio of the change in volume to its original volume, is called as volumetric strain.

$$e = \frac{\Delta v}{v}$$ → here, Δv → change in volume

* <u>Hooke's Law</u>: (most imp) →2 marks regular question

It states that "when a material is loaded such that the intensity of stress within a certain limit, the ratio of the intensity of the stress to the corresponding strain is a constant."

$$\frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e} = \text{constant}.$$

$$\Rightarrow \frac{\sigma}{e} = \text{constant}$$

$$\Rightarrow \sigma = \text{constant} \times e$$

$$\Rightarrow \boxed{\sigma \propto e}$$

→ It also can be stated as "Stress is directly proportional to strain, within a certain limit."

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* Modulus of Elasticity (or) Young's Modulus:

In case of axial loading, the ratio of the intensity of tensile or compressive stress to the corresponding strain is constant.

→ This ratio is called modulus of elasticity or young's modulus and is denoted by 'E'.

$$\frac{\sigma}{e} = \text{constant} = E$$

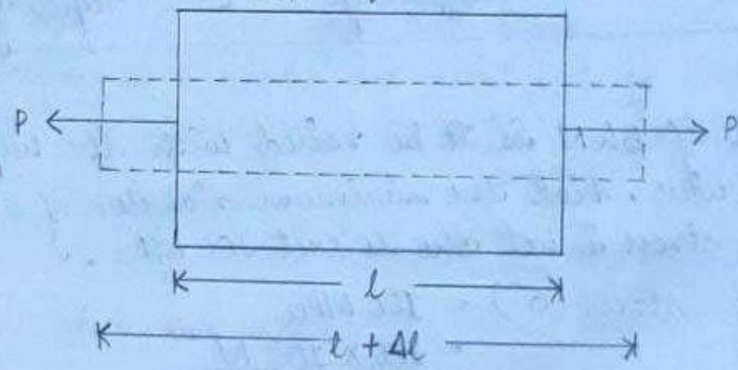
[∵ constant is nothing but the young's modulus 'E'.]

$$\Rightarrow \boxed{E = \frac{\sigma}{e}}$$

* Deformation of prismatic bar due to uni-axial load:

(prismatic → throughout the length no change in dimension or no change in cross-sectional area.)

(uni-axial → 1 or same.)



Consider a prismatic bar is subjected to axial tensile load 'P' → same as an external force.

The stress will be introduced due to load P is given by,

$$\boxed{\sigma = \frac{P}{A}}$$

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we know, strain : $e = \frac{\Delta l}{l}$

According to Hooke's law,

$\sigma \propto e$

$\Rightarrow \sigma = Ee$

$\Rightarrow \frac{P}{A} = E \frac{\Delta l}{l}$ $\left[\because \sigma = \frac{P}{A} \text{ and } e = \frac{\Delta l}{l} \right]$

*** $\Rightarrow \boxed{\Delta l = \frac{Pl}{AE}}$

→ In the above formula, the term 'AE' is known as <u>axial rigidity</u>. [AE → axial rigidity]

<u>Questions</u>:

① A load of 5kN is to be raised with the help of a steel wire. Find the minimum diameter of a wire, if the stress is not ~~the~~ to exit 100 MPa.

→ <u>given</u>: stress (σ) = 100 MPa

$= 100 \times 10^6 \frac{N}{m^2}$.

external force (P) = 5kN

$= 5 \times 10^3 N$

$$\sigma = \frac{P}{A}$$

$$\Rightarrow 100 \times 10^6 = \frac{5 \times 10^3}{\frac{\pi}{4} d^2}$$

$$\Rightarrow d^2 = \frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}$$

$$\Rightarrow d = \sqrt{\frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}}$$

$$\Rightarrow \boxed{d = 7.97 \times 10^{-3} m}$$

$$\Rightarrow d = 7.97 \times 10^{-3} \times 10^3 \text{ mm}$$

$$\Rightarrow \boxed{d = 7.97 \text{ mm}}$$

(2) A steel rod 500 mm long and 20 mm × 10 mm in cross-section is subjected to axial pull of 300 kN. If modulus of elasticity is $2 \times 10^5 \frac{N}{mm^2}$. Calculate the elongation of the rod.
Also calculate the strain induced in the bar.

→ given: $l = 500$ mm

$P = 300 kN = 300 \times 10^3 N$

$E = 2 \times 10^5 N/mm^2$

$A = 20 mm \times 10 mm$
$= 200 mm^2$

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∴ Δl

$$\therefore \Delta l = \frac{Pl}{AE} = \frac{300 \times 10^3 \times 500}{200 \times 2 \times 10^5} = 3.75\ mm$$

$$\text{and strain} = \frac{\Delta l}{l} = \frac{3.75}{500} = 7.5 \times 10^{-3}.$$

② A hollow cylinder 2m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in cylinder, also find deformation of the cylinder. Take $E = 100\ GPa$.

→ given: $l = 2m = 2 \times 1000\ mm = 2000\ mm$

$D = 50\ mm$

$d = 30\ mm$

$E = 100\ GPa$

$P = 25\ kN = 25 \times 10^3\ N$

∴ Area of hollow cylinder : $A = \frac{\pi}{4}(D^2 - d^2)$

$\Rightarrow A = \frac{\pi}{4}((50)^2 - (30)^2)$

$\Rightarrow \boxed{A = 1256\ mm^2}$

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$$E = 100\ GPa$$
$$= 100 \times 10^9 \ \frac{N}{m^2}$$
$$= 100 \times 10^9 \ \frac{N}{10^6}$$
$$= 100 \times 10^9 \times 10^6 \ \frac{N}{mm^2}$$
$$E = 100 \times 10^3 \ \frac{N}{mm^2}$$

$1m \rightarrow 10^3 mm$
$1m^2 \rightarrow (10^3)^2 mm^2$
$1m^2 = 10^6 mm^2$

$$\therefore \sigma = \frac{P}{A} = \frac{25 \times 10^3}{1256} = 19.90 \ \frac{N}{mm^2}$$

$$\text{and } \Delta l = \frac{Pl}{AE} = \frac{25 \times 10^3 \times 2000}{1256 \times 100 \times 10^3} = 0.398 \ mm.$$

* POISSON'S RATIO $\left(\mu / \frac{1}{m}\right)$

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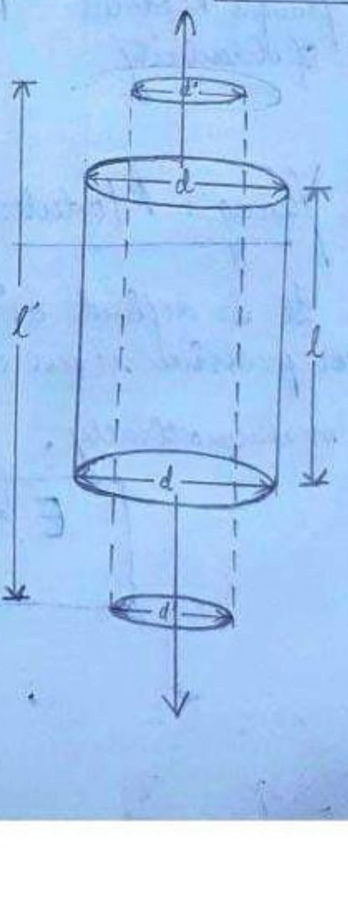
linear strain $= + \frac{l'}{l}$

lateral strain $= - \frac{d'}{d}$



→ Poisson's ratio is defined as "the ratio between the lateral strain to the longitudinal strain."

→ It is denoted as μ or $\frac{1}{m}$ and the value of μ is always less than 1.

mathematically,

$$\mu = \frac{-\text{lateral strain}}{\text{longitudinal strain}}$$

→ It is a constant and it has no unit.

→ ex: (i) Poisson ratio of concrete = 0.15 to 0.3
(ii) Poisson ratio of steel = 0.27 to 0.3.

→ Poisson ratio varies from **−1 to 0.5**

Elastic Constant

- E: Young's Modulus of Elasticity
- K: Bulk Modulus
- C: Shear Modulus (or) Modulus of Rigidity

(i) Young's Modulus (E):

It is defined as "the ratio of tensile or compressive stress to the corresponding strain."

mathematically,

$$E = \frac{\sigma}{e}$$

(ii) Bulk Modulus (K) :

when an uniform element is subjected to equal stress in 3 mutually perpendicular directions then "the ratio of direct stress to the volumetric strain is known as bulk modulus."

mathematically,

$$K = \frac{\sigma}{e_v}$$

$\left[\because e_v = \frac{\Delta V}{V} \right]$ volumetric strain

(iii) Shear Modulus (C or G) :

It is defined as "the ratio of shear stress to the shear strain."

→ Higher the value of shear modulus, shows that the body is highly rigid.

mathematically,

$$C = \frac{\tau}{\epsilon}$$

where, τ → shear stress

ϵ → shear strain

* Relationship between Elastic Constants E, K, C:

(i) Relationship between E and K:

$$E = 3K\left(1 - \frac{2}{M}\right)$$

(ii) Relationship between E and C:

$$E = 2C\left(1 + \frac{1}{m}\right)$$

where, $\frac{1}{m}$ is poisson's ratio

(iii) Relationship between E, K and C:

$$E = \frac{9KC}{3K + C}$$

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* **Proof:** $$E = \frac{9KC}{3K+C}$$

we know,

$$E = 3K\left(1 - \frac{2}{M}\right) \Rightarrow 1 - \frac{2}{M} = \frac{E}{3K} \quad \text{—— (1)}$$

$$E = 2C\left(1 + \frac{1}{m}\right) \Rightarrow 1 + \frac{1}{m} = \frac{E}{2C} \quad \text{—— (2)}$$

multiplying 2 in eq^n (2)

$$2\left(1 + \frac{1}{M}\right) = 2\left(\frac{E}{2C}\right)$$

$$\Rightarrow 2 + \frac{2}{M} = \frac{E}{C} \quad \text{—— (3)}$$

now adding (1) and (3)

$$1 - \frac{2}{M} + 2 + \frac{2}{M} = \frac{E}{3K} + \frac{E}{C}$$

$$\Rightarrow 1 + 2 = \frac{E}{3K} + \frac{E}{C}$$

$$\Rightarrow 3 = \frac{EC + E3K}{3KC}$$

$$\Rightarrow 3(3KC) = EC + E3K$$

$$\Rightarrow 9KC = E(C + 3K)$$

$$\Rightarrow \boxed{E = \frac{9KC}{3K + C}}$$ hence proved.

Questions:

① The modulus of rigidity of a material is $0.8 \times 10^5 \frac{N}{mm^2}$. find the poisson's ratio if the modulus of elasticity of that material is $2.1 \times 10^5 \frac{N}{mm^2}$.

→ given: $C = 0.8 \times 10^5 \frac{N}{mm^2}$

$E = 2.1 \times 10^5 \frac{N}{mm^2}$

μ or $\frac{1}{M} = ?$

$$\therefore E = 2C\left(1 + \frac{1}{M}\right)$$

$$\Rightarrow 2.1 \times 10^5 \frac{N}{mm^2} = 2 \times 0.8 \times 10^5 \frac{N}{mm^2}\left(1 + \frac{1}{M}\right)$$

$$\Rightarrow \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} = 1 + \frac{1}{M}$$

$$\Rightarrow \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1 = \frac{1}{M}$$

$$\Rightarrow \frac{1}{M} \text{ or } \mu = \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1.$$

$$\Rightarrow \boxed{\mu = 0.3125} \text{ - Ans}$$

② The modulus of rigidity of material is $0.8 \times 10^5 \frac{N}{mm^2}$ when a 6mm × 6mm rod of this material was subjected to an axial pull of 3600 N. It was found that the lateral dimension of the rod changed to 5.9991 mm × 5.9991 mm. Find the poisson's ratio and modulus of elasticity of that rod.

→ given: $C = 0.8 \times 10^5 \frac{N}{mm^2}$.

$A = 6\,mm \times 6\,mm$
$= 36\,mm^2$

$P = 3600\,N$

$$\sigma = \frac{P}{A} = \frac{3600}{36} = 100 \frac{N}{mm^2}$$

longitudinal strain, $E = \frac{\sigma}{e} \rightarrow \boxed{e = \frac{\sigma}{E}}$

$$\text{lateral strain} = \frac{\text{change in lateral dimension}}{\text{original dimension}}$$

$$= \frac{6 - 5.9991}{6} = 0.00015$$

Poisson's ratio:

$$\frac{1}{M} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\Rightarrow \frac{1}{M} = \frac{0.00015}{\frac{\sigma}{E}}$$

$$\Rightarrow \frac{1}{M} = \frac{0.00015}{100} \times E$$

$$\Rightarrow \frac{1}{EM} = \frac{0.00015}{100}$$

$$\Rightarrow ME = \frac{100}{0.00015}$$

$$\Rightarrow \boxed{ME = \frac{2 \times 10^6}{3}} \quad \text{—— (1)}$$

we know, $E = 2C\left(1 + \frac{1}{M}\right)$

$$\Rightarrow E = 2C \frac{(M+1)}{M}$$

$$\Rightarrow ME = 2C (M+1)$$

$$\Rightarrow \boxed{ME = 2 \times 0.8 \times 10^5 (M+1)} \quad \text{—— (2)}$$

equating eqⁿ (1) and (2)

$$\Rightarrow 2 \times 0.8 \times 10^5 (M+1) = \frac{2 \times 10^6}{3}$$

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$$\Rightarrow M + 1 = \frac{2 \times 10^6}{3(2 \times 0.8 \times 10^5)}$$

$$\Rightarrow M = \frac{2 \times 10^6}{3(2 \times 0.8 \times 10^5)} - 1$$

$$\Rightarrow \boxed{M = 3.167}$$

$$\because \mu = \frac{1}{M} = \frac{1}{3.167}$$

$$\Rightarrow \boxed{\mu = 0.315}$$

$$\therefore E = 2C\left(1 + \frac{1}{M}\right)$$

$$\Rightarrow E = 2 \times 0.8 \times 10^5 (1 + 0.315)$$

$$\Rightarrow E = 210400$$

$$\Rightarrow \boxed{E = 2.1 \times 10^5 \frac{N}{mm^2}}$$

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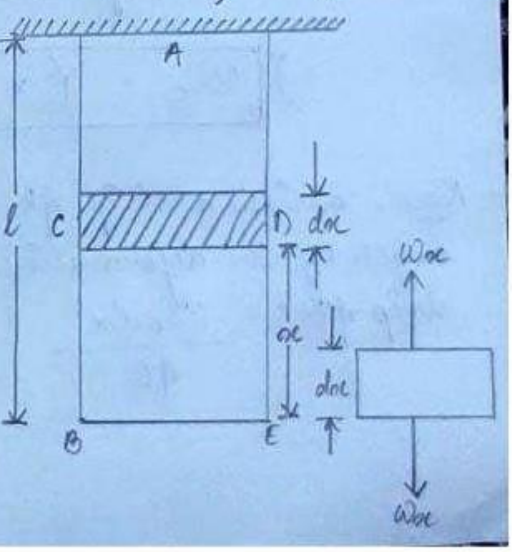
* Axial deformation of bar due to its self-weight:

Considering a prismatic bar of uniform cross-sectional of length 'l' suspended freely.

weight of the bar = W

unit weight of the bar = γ



$$\left[\gamma = \frac{W}{\text{volume}} \Rightarrow \gamma = \frac{W}{Ax}\right]$$

$A \rightarrow$ area

$x \rightarrow$ length

$Ax \rightarrow$ volume

Assuming cross-sectional area = 'A'

Consider a small element of length 'dx' at a distance 'x' from the free end.

The element is subjected to weight of bar below element (i.e. BCDE)

weight of bar below element dx

$$\Rightarrow W_x = \gamma (Ax)$$

Elongation of elemental length dx

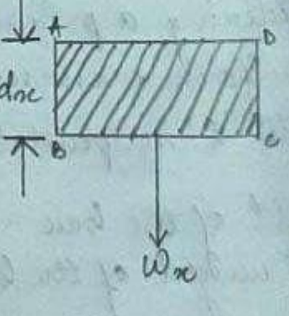
$$\Rightarrow \boxed{d\Delta = \frac{W_x \, dx}{AE}}$$

Let E, ρ, γ, be the Young's Modulus, density and unit weight of the body respectively.

we know, (volume) $V_x = A \times x$

$$\gamma = \frac{W_x}{V_x}$$

$$\Rightarrow \boxed{W_x = \gamma \times V_x}$$



Now, consider a strip ABCD of length by the deformation for the strip ABCD = $\frac{W_x \, dx}{AE}$

Total deformation of the bar $= \int_0^l \frac{w_x \times dx}{AE}$

$$= \int_0^l \frac{Y \times V_x}{AE} dx$$

$$= \int_0^l \frac{Y \times A \times x}{AE} dx$$

$$= \int_0^l \frac{Yx}{E} dx$$

$$= \frac{Y}{E} \int_0^l x \, dx \quad \left[\because Y \text{ and } E \text{ are constants}\right]$$

$$= \frac{Y}{E} \left(\frac{x^2}{2}\right)_0^l$$

$$= \frac{Y}{E} \left[\frac{l^2}{2} - \frac{0^2}{2}\right]$$

$$= \frac{Y}{E} \times \frac{l^2}{2}.$$

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$$\Rightarrow \boxed{\Delta = \frac{\gamma l^2}{2E}}$$

$\Delta l \rightarrow$ total deformation of the bar.

* **NOTES**:

① Ultimate stress = $\frac{\text{maximum load}}{\text{Area}}$

② Yield stress = $\frac{\text{yield point load}}{\text{Area}}$

③ Safe stress = $\frac{\text{yield stress}}{\text{load factor}}$

Questions:

① A following data refers to a mild steel specimen tested in a laboratory:

(i) diameter of specimen = 25 mm
(ii) length of specimen = 300 mm
(iii) extension under a load 50kN = 0.045 mm
(iv) load at yield point = 127.65 kN
(v) maximum load = 208.60 kN
×(vi) length of the specimen after failure = 375 mm
×(vii) neck diameter = 17.75 mm

Now, determine: (i) Young's Modulus
(ii) Yield point stress
(iii) Ultimate stress
(iv) safe stress adopting a factor of safety of 2.

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→ given : $d = 25\ mm$

$l = 300\ mm$

$P = 50\ kN$

$= 50 \times 10^3\ N$

$\Delta l = 0.045\ mm$

yield point load = $127.65\ kN$

$= 127.65 \times 10^3\ N$

maximum load = $208.60\ kN$

$= 208.60 \times 10^3\ N$

load factor = 2.

(i) $\Delta l = \frac{Pl}{AE}$

$\Rightarrow E = \frac{Pl}{A \Delta l}$

$\Rightarrow E =$

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(ii) Yield point stress = $\frac{\text{yield point load}}{\text{area}}$

$$= \frac{127.65 \times 10^3 \ N}{\frac{\pi}{4}(25)^2 \ mm^2}$$

$$= 260.05 \ \frac{N}{mm^2}$$

(iii) Ultimate stress = $\frac{\text{maximum load}}{\text{area}}$

$$= \frac{208.60 \times 10^3 \ N}{\frac{\pi}{4}(25)^2 \ mm^2}$$

$$= 424.96 \ \frac{N}{mm^2}.$$

(iv) Safe stress = $\frac{\text{yield stress}}{\text{load factor}}$

$$= \frac{260.05}{2} \ \frac{N}{mm^2}.$$

$$= 130.025 \ \frac{N}{mm^2}.$$

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Mechanical Properties Of Materials

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MECHANICAL PROPERTIES OF MATERIALS :

① Rigidity:

(i) This is defined as the property possessed by a solid body to change its shape.

(ii) It means when an external force is applied to the solid material, there won't be any change in its shape due to intermolecular attraction by the closely packed particles.

(iii) This is the property of the material to resistance the bending.

② Elasticity:

(i) This is the property of a body by virtue of which in return its original shape after removal of external force causing deformation which is applied on it.

(ii) Elastic property entirely depends upon the type of material and not on the shape and size.

③ Plasticity:

(i) This is the ability of a solid material to undergo permanent deformation when force is applied to it.

(ii) The ability of a material to retain the changed shape under application of load is known as plasticity.

(iii) Plastic deformation is the property of ductile and malleable solids.

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④ Compressibility:

(i) This is the property of material by virtue of which it tends to flatten and reduce in size under pressure.

(ii) This nature or property of material changes the inter-molecular structure of the material.

⑤ Hardness:

(i) The property of material by virtue of which it resists the local surface deformation when undergoes abrasion, drilling, impact, etc.

(ii) It is the state of material, being hard for which it can withstand friction.

⑥ Toughness:

(i) The amount of energy per unit volume that a material can absorb before rupture is called toughness.

(ii) It can be defined as the ability of a material to resist breaking when force is applied to it.

(iii) This property allows the material to deform before rupture or fracture.

⑦ Stiffness:

(i) The property of material which resists deformation when a force is applied to it. This is the rigidity of a material.

(ii) The material having more flexibility has less stiffness.

(iii) A stiff material has high Young modulus.

⑧ Brittleness:

(i) The property of a material by which it fractures when subjected to stress without deformation.

(ii) Brittle material has a little tendency to deform before rupture.

(iii) It has small plastic region.

Ex: Bone, concrete, ceramic, cast Iron, glass products, etc.

⑨ Ductility:

(i) It is defined as the ability of a material to undergo permanent deformation through elongation and reduction in cross-sectional area or bending at room temperature without fracturing.

(ii) This ~~or~~ is an ability to undergo vast permanent deformation in tension.

Example of ductile material → copper, aluminium, steel.

★ Opposite to ductile is brittle.

⑩ Malleability:

(i) This is the property of material by which it can be beaten to form its thin sheets.

Ex: lead, tin, gold, silver, aluminium, copper, Iron.

⑪ Creep:

(i) This is the permanent change in shape and size of a material which increases as a function of time under application of load and elevated temperature.

(ii) Creep is time independent.

(iii) Creep begins at different temperature for different material.

⑫ Fatigue:

(i) This is the deterioration of the material subjected to repeated cycle of stress and strain resulting in progressive cracking, eventually producing fracture.

(ii) This is the weakening of a material caused by cyclic loading that results in progressive damage and the growth of cracks.

(iii) This is responsible for 90% of mechanical failure.

⑬ Durability:

(i) The ability of a material to remain serviceable during the useful time without damaging the material.

(ii) It represents how long the material works.

⑭ Tenacity:

→ The property of material to resist the breaking is known as tenacity.

★ NOTES:

① Percentage of Elongation:

(a) It is a measure of ductility.

(b) This can be obtained as $= \frac{\text{final length} - \text{initial length}}{\text{initial length}} \times 100$

or $\frac{\Delta l}{l} \times 100$

where, Δl = change in length

② Percentage of reduction in area:

ⓐ This is the measure of the specimen that how much that specimen narrowed when it undergoes some load application.

ⓑ It is obtained as follows:

$$\frac{\text{final area} - \text{initial are}}{\text{initial area}} \times 100$$

*** Signification of Percentage Elongation and Percentage Reduction in area:**

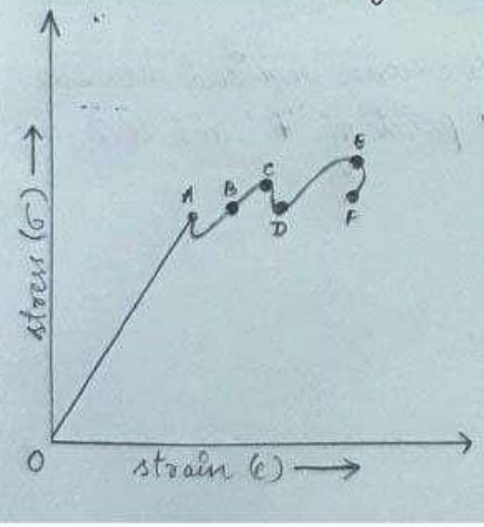
(i) Percentage elongation is a property of material which ~~prode~~ provides a value of its ductility.

(ii) The greater deformation before breaking shows the material is more ductile.

(iii) Percentage elongation is a measure of ductility.

(iv) Percentage reduction area is also a measure of ductility.

*** Stress - Strain diagram for mild steel: (Imp)**



→ This curve is obtained when a mild steel specimen undergoes a tensile test. The plot from O to A is a straight line, this portion obeys Hooke's law and the straight line is called the limit of proportionality.

→ In this range of extension, the stress is proportional to strain i.e. $\boxed{\sigma \propto e}$

→ If the specimen is extended beyond the limit of proportionality upto the point 'B', the material still remains elastic.

→ But from A to B, stress-strain relation is not linear.

→ The stress at 'B' is called <u>elastic limit</u>.

→ Again if the specimen is extended beyond the elastic limit, plastic deformation occurs.

→ In the range of 'B' to 'C' strain increases without increase in stress.

→ At point 'C' the material goes extended with decrease in load and the stress at point C is called <u>upper-yield point</u>.

→ At point 'D' the material again offers resistance to greater extension and the stress corresponding to this point is called <u>lower-yield point</u>.

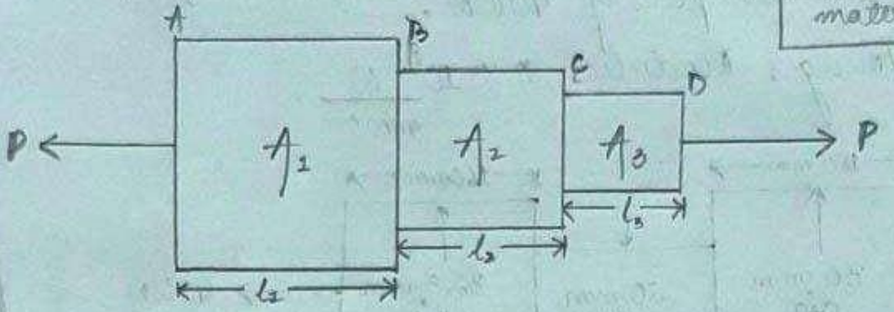
→ As the load is increased, the extension increases and the point 'E' indicates the necking of the specimen and the stress corresponding to this point is called <u>ultimate tensile stress</u>.

→ As the extension is increased, the load required decreases and the specimen breaks at the point of 'F' and this point is called <u>stress failure</u>.

* Elongation in bar of varying section: 20.11.2021

changes in cross section

E is same, material to material



The elongation for the portion AB : $\Delta l_1 = \frac{P l_1}{A_1 E}$

The elongation for the portion BC : $\Delta l_2 = \frac{P l_2}{A_2 E}$

The elongation for the portion CD : $\Delta l_3 = \frac{P l_3}{A_3 E}$

So total elongation : $\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$

$$= \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

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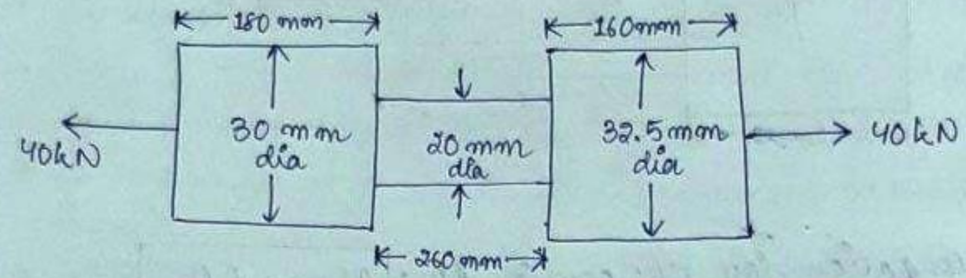
① A bar consisting of 3 lengths. Find the stresses in the three parts and the total elongation of the bar for an axial pull of 40kN.

Take Young's Modulus $2 \times 10^5 \frac{N}{mm^2}$.



→ given: $P = 40kN = 40 \times 10^3 N$

$E = 2 \times 10^5 \frac{N}{mm^2}$

$A_1 = \frac{\pi}{4}(30)^2 = 706.85 \ mm^2$

$A_2 = \frac{\pi}{4}(20)^2 = 314.15 \ mm^2$

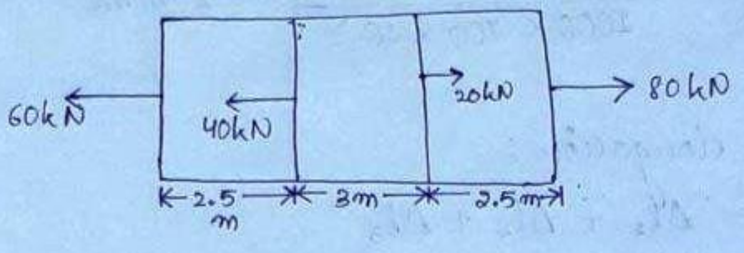
$A_3 = \frac{\pi}{4}(32.5)^2 = 829.579 \ mm^2$

$\Delta l_1 = \frac{40 \times 10^3 \times 180}{706.85 \times 2 \times 10^5} = 0.50 \ mm$

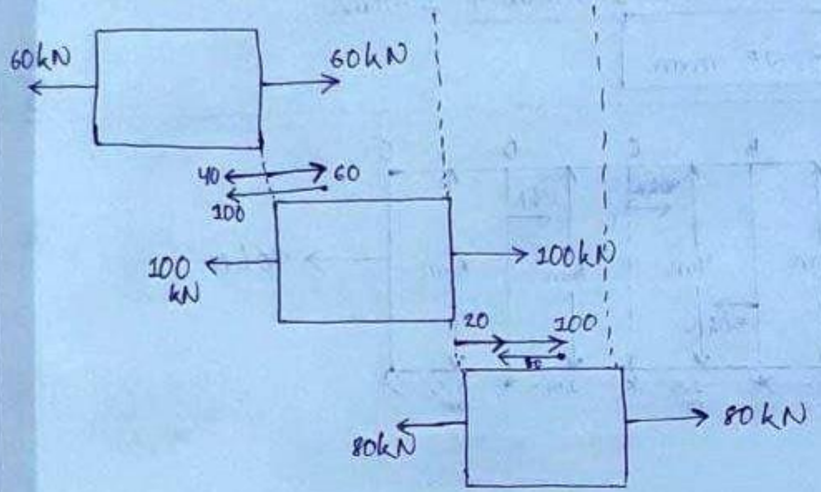
$\Delta l_2 = \frac{40 \times 10^3 \times 260}{314.15 \times 2 \times 10^5} = 0.165 \ mm$

$\Delta l_3 = \frac{40 \times 10^3 \times 160}{829.579 \times 2 \times 10^5} = 0.038 \ mm$

③ A steel rod ABCD of 1000 mm^2 cross sectional area and 8m long is subjected to forces as shown in the figure below. Find the total deformation if young's modulus of the bar is 200 GPa. All forces are in kN.



FBD:



$P_1 = 60kN = 60 \times 10^3$ N

$P_2 = 100kN = 100 \times 10^3$ N

$P_3 = 80kN = 80 \times 10^3$ N

$l_1 = 2.5m = 2.5 \times 10^3$ mm

$l_2 = 3m = 3 \times 10^3$ mm

$l_3 = 2.5m = 2.5 \times 10^3$ mm

$E = 200 GPa$ $\quad A = 1000\ mm^2$

$= 200 \times 10^9 \frac{N}{m^2}$

$= 200 \times 10^9 \frac{N}{10^6 mm^2}$

$= 200 \times 10^{9-6} \frac{N}{mm^2}$

$= 200 \times 10^3 \frac{N}{mm^2}$

$$\Delta l_1 = \frac{P_1 l_1}{AE} = \frac{60 \times 10^3 \times 2.5 \times 10}{1000 \times 200 \times 10^3} = 0.75 \text{ mm}$$

$$\Delta l_2 = \frac{P_2 l_2}{AE} = \frac{100 \times 10^3 \times 3 \times 10^3}{1000 \times 200 \times 10^3} = 1.5 \text{ mm}$$

$$\Delta l_3 = \frac{P_3 l_3}{AE} = \frac{80 \times 10^3 \times 2.5 \times 10^3}{1000 \times 200 \times 10^3} = 1 \text{ mm}$$

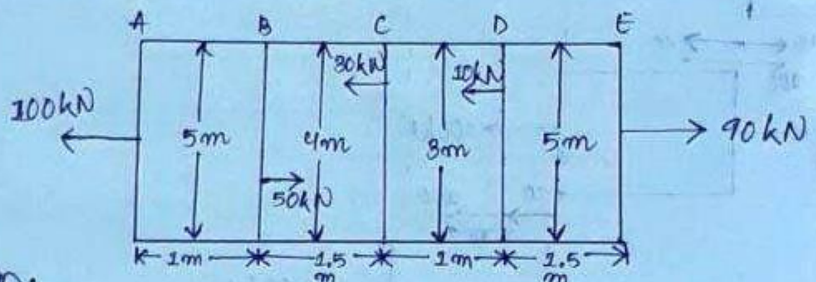
∴ Total elongation :

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

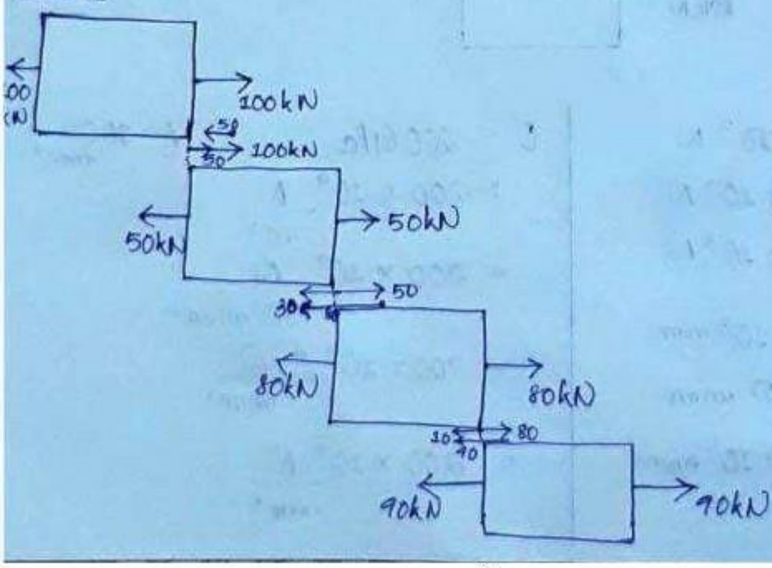
$$\Rightarrow \Delta l = 0.75 \text{mm} + 1.5 \text{mm} + 1 \text{ mm}$$

$$\Rightarrow \boxed{\Delta l = 3.25 \text{ mm}}$$

(4)



FBD:



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$P_1 = 100\,kN = 100\times 10^3\,N$ | $l_1 = 1\,m$ | $d_1 = 5\,m$

$P_2 = 50\,kN = 50\times 10^3\,N$ | $l_2 = 1.5\,m$ | $d_2 = 4\,m$

$P_3 = 80\,kN = 80\times 10^3\,N$ | $l_3 = 1\,m$ | $d_3 = 3\,m$

$P_4 = 90\,kN = 90\times 10^3\,N$ | $l_4 = 1.5\,m$ | $d_4 = 5\,m$

$A_1 = \frac{\pi}{4}(5)^2 = 19.63\,m^2$

$A_2 = \frac{\pi}{4}(4)^2 = 12.56\,m^2$

$A_3 = \frac{\pi}{4}(3)^2 = 7.06\,m^2$

$A_4 = \frac{\pi}{4}(5)^2 = 19.63\,m^2$

$E = 2.5\,MPa = 2.5\times 10^6\,\frac{N}{m^2}$

$$\Delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{100\times 10^3\times 1}{19.63\times 2.5\times 10^6} = 2.037\times 10^{-3}\,m$$

$$\Delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{50\times 10^3\times 1.5}{12.56\times 2.5\times 10^6} = 2.38\times 10^{-3}\,m$$

$$\Delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{80\times 10^3\times 1}{7.06\times 2.5\times 10^6} = 4.53\times 10^{-3}\,m$$

$$\Delta l_4 = \frac{P_4 l_4}{A_4 E} = \frac{90\times 10^3\times 1.5}{19.63\times 2.5\times 10^6} = 2.75\times 10^{-3}\,m$$

∴ Total elongation:

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4$$

$$\Rightarrow \Delta l = (2.037\times 10^{-3}) + (2.38\times 10^{-3}) + (4.53\times 10^{-3}) + (2.75\times 10^{-3})\,m$$

$$\Rightarrow \Delta l = 0.011697\,m$$

$$\Rightarrow \boxed{\Delta l = 11.697\,mm}$$

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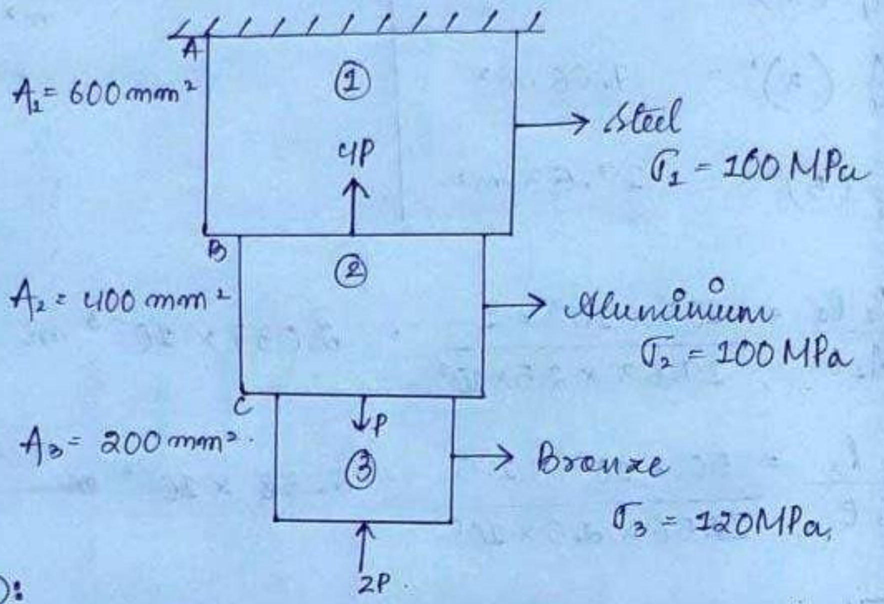
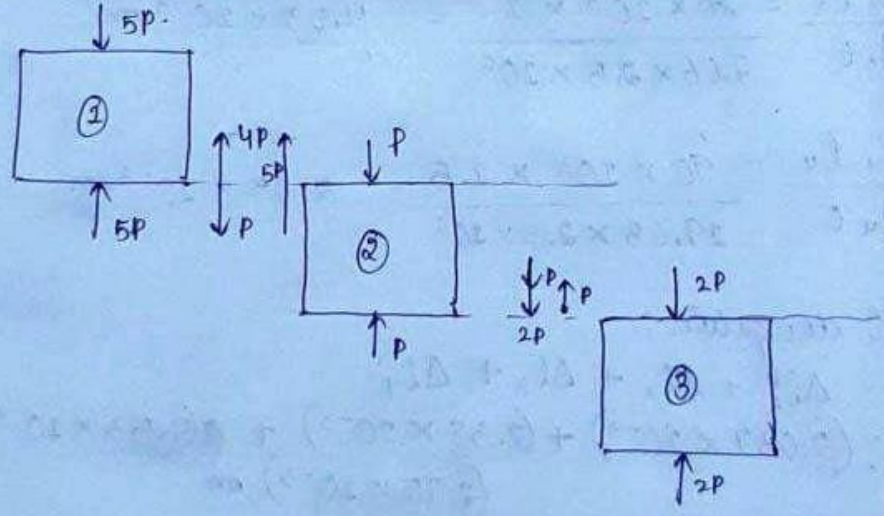
⑤ An aluminium bar is rigidly attached between steel bar and a bronze bar shown in figure. Axial forces are applied at the position indicated. Determine the maximum value of P that will not exceed stress of 160 MPa in steel, 100 MPa in aluminium and 120 MPa in bronze.



FBD:



$\sigma_1 = 160 \text{ MPa} = 160 \times 10^6 \frac{N}{m^2} = 160 \times \cancel{10^6} \frac{N}{\cancel{10^6} mm^2}$

$\Rightarrow \boxed{\sigma_1 = 160 \frac{N}{mm^2}}$

$\sigma_2 = 100 \text{ MPa} = 100 \times 10^6 \frac{N}{m^2} = 100 \times \cancel{10^6} \frac{N}{\cancel{10^6} mm^2}$

$\Rightarrow \boxed{\sigma_2 = 100 \frac{N}{mm^2}}$.

$\sigma_3 = 120 \text{ MPa} = 120 \times 10^6 \frac{N}{m^2} = 120 \times \cancel{10^6} \frac{N}{\cancel{10^6} mm^2}$

$\Rightarrow \boxed{\sigma_3 = 120 \frac{N}{mm^2}}$

$A_1 = 600 \text{ mm}^2$ $\quad P_1 = 5P$

$A_2 = 400 \text{ mm}^2$ $\quad P_2 = P$

$A_3 = 200 \text{ mm}^2$ $\quad P_3 = 2P$.

∴ we know that :

For section ① :

$$\sigma_1 = \frac{P_1}{A_1}$$

$$\Rightarrow 160 = \frac{5P}{600} \Rightarrow P_1 = \frac{160 \times 600}{5}$$

$$\Rightarrow \boxed{P_1 = 19200 \text{ N}}$$

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For section ② :

$$\sigma_2 = \frac{P_2}{A_2}$$

$$\Rightarrow 200 = \frac{P}{400} \Rightarrow P_2 = 100 \times 400$$

$$\Rightarrow \boxed{P_2 = 40000\ N}$$

For section ③ :

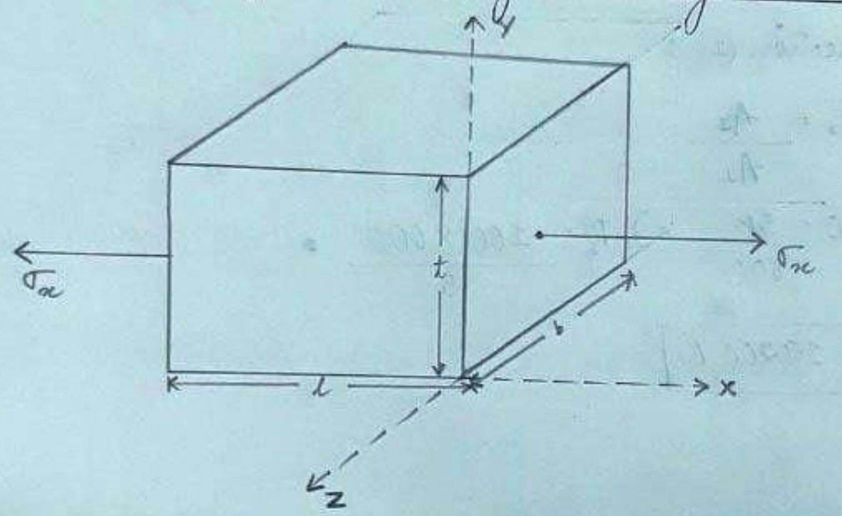
$$\sigma_3 = \frac{P_3}{A_3}$$

$$\Rightarrow 120 = \frac{2P}{200} \Rightarrow P_3 = \frac{120 \times 200}{2}$$

$$\Rightarrow \boxed{P_3 = 12000\ N}$$

27. 11. 2021

① Volumetric strain of a Rectangular Section Under the action of a stress along the longitudinal strain:



let l, b, t are the length, breadth and thickness of the rectangular block.

σ_x = tensile stress in x-direction

E = Young's Modulus

μ = Poisson's ratio

$$= \frac{-\text{ latural strain}}{\text{longitudinal strain}}$$

longitudinal strain : $e_x = \dfrac{\sigma_x}{E}$

$$\mu = -\frac{e_y}{e_x}$$

$$\Rightarrow e_y = -\mu \cdot e_x$$

$$\Rightarrow e_y = -\mu \frac{\sigma_x}{E} \qquad \left[\because e_x = \frac{\sigma_x}{E}\right]$$

Similarly, $e_z = -\mu \dfrac{\sigma_x}{E}$

But we know that,

$$e_x = \frac{\delta l}{l}, \quad e_y = \frac{\delta b}{b}, \quad e_z = \frac{\delta t}{t}$$

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we know that,

volume of a rectangular (3D) block is given by:

$$V = l \times b \times t$$

Now, derivating both the sides:

$$\frac{dv}{v} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}$$

$$\left[\because \frac{d}{l}(l \times b \times t) + \frac{d}{b}(l \times b \times t) + \frac{d}{t}(l \times b \times t) = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t} \right]$$

$$\Rightarrow e_v = e_x + e_y + e_z$$

$$\left[\because e_v = \frac{dv}{v} = \text{volumetric strain} \quad e_x = \frac{dl}{l} \quad e_z = \frac{dt}{t} \quad e_y = \frac{db}{b} \right]$$

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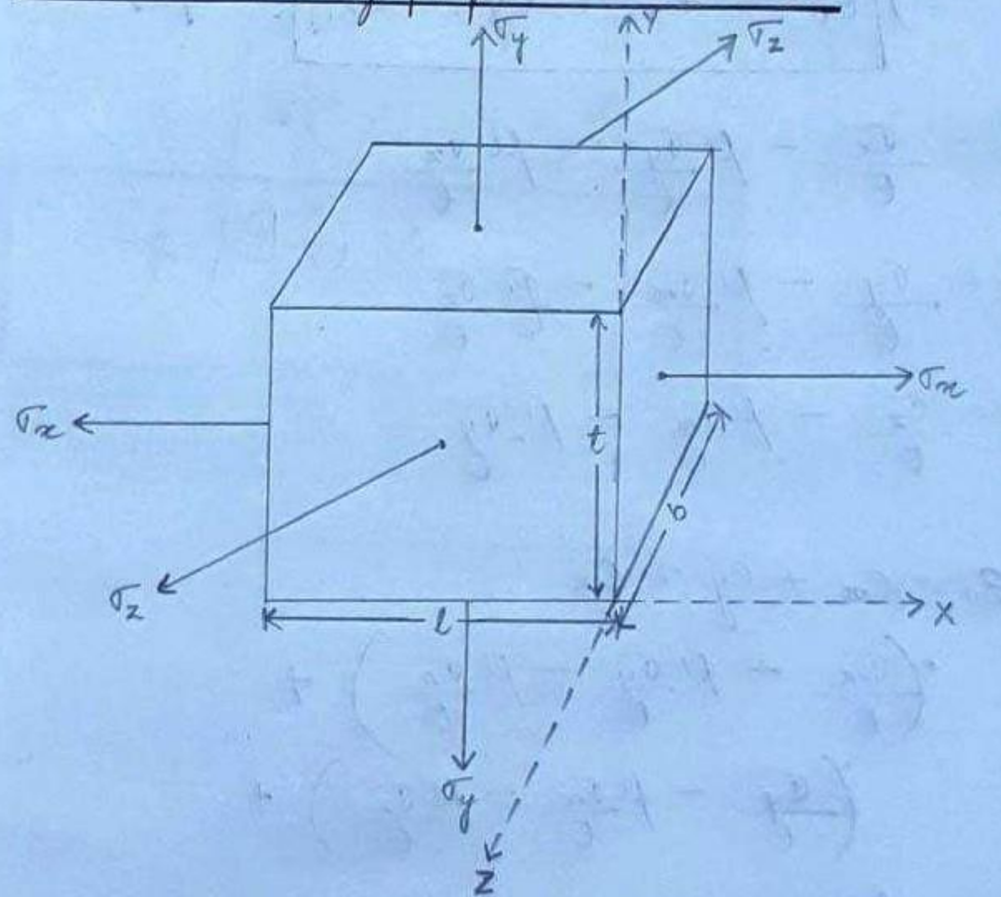
Now put the value of e_x, e_y and e_z

$$e_v = \frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E}$$

$$\Rightarrow e_v = \frac{\sigma_x}{E} - 2\mu\frac{\sigma_x}{E}$$

$$\Rightarrow \boxed{e_v = \frac{\sigma_x}{E}(1-2\mu)}$$ ★★★★★

② **Volumetric strain of a rectangular block subjected to three mutually perpendicular stress:**



Let l, b, t are the length, breadth and thickness of the rectangular block.

σ_x, σ_y, σ_z are the three stresses act in the direction of x, y, z.

We know that,

$$V = l \times b \times t.$$

and $\frac{dv}{v} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}$

$$\Rightarrow \boxed{e_v = e_x + e_y + e_z}$$

- $e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}.$
- $e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$
- $e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}.$

$\therefore e_v = e_x + e_y + e_z$

$$= \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right)$$

$$\Rightarrow e_v = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E} + \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - \mu\sigma_y - \mu\sigma_z + \sigma_y - \mu\sigma_x - \mu\sigma_z - \sigma_z - \mu\sigma_x - \mu\sigma_y\right]$$

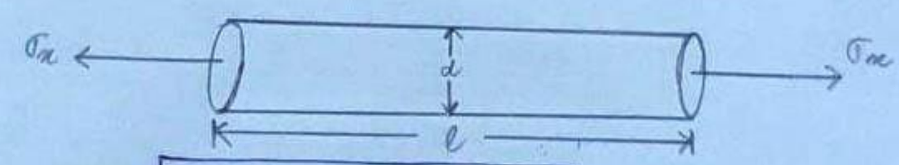
$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - \mu\sigma_x - \mu\sigma_x + \sigma_y - \mu\sigma_y - \mu\sigma_y + \sigma_z - \mu\sigma_z - \mu\sigma_z\right]$$

$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - 2\mu\sigma_x + \sigma_y - 2\mu\sigma_y + \sigma_z - 2\mu\sigma_z\right]$$

$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x(1-2\mu) + \sigma_y(1-2\mu) + \sigma_z(1-2\mu)\right]$$

$$\Rightarrow \boxed{e_v = \frac{1}{E}\left[(1-2\mu)(\sigma_x + \sigma_y + \sigma_z)\right]}$$

③ Volumetric strain for a circular Rod:



$$\boxed{e_v = 2e_d + e_l}$$

where, e_v = volumetric strain

e_d = lateral strain

e_l = longitudinal strain

Complex Stress and Strain .

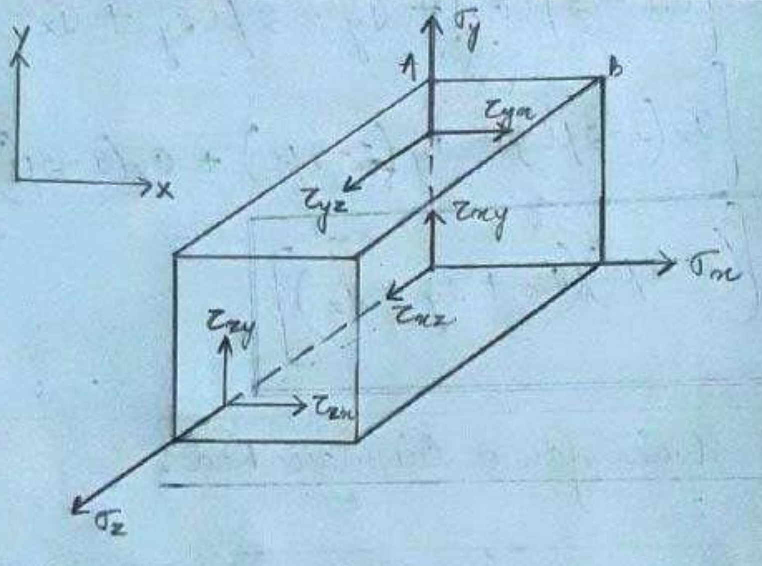
(4) Volumetric strain for a sphere:

$$e_v = 3\,e_d$$

CHAPTER-3

29.11.2021

COMPLEX STRAIN AND STRESS:



3-D representation of stresses

(or)

3-dimensional stress system